Based on collaboration with Massimo Taronna [arXiv:1311.0242]

CUBIC-INTERACTION INDUCED DEFORMATIONS HIGHER-SPIN SYMMETRY

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Field theories describing H6s3less HS ?

- massless spin $1 \rightarrow Gauge theory$ (internal symmetry)
- massless spin $2 \rightarrow Gravity$ (isometry)
- massless spin $S \rightarrow ??$ (Unbroken symmetry of ST ?

<u>Recent Intro. / Review :</u> [0503128] (Vasiliev eq.) [1007.0435] (General review) [1112.4285] (ST aspect)



<u>Review :</u> [1208.4036], [1208.5182]

HS theory in AdS

CFT on the boundary

AdS/CFT

- 3d Vector O(N)
- WN minimal model

Vasiliev eq. in AdS4
PV eq. in AdS3

Free theory: possible (up to a few subtleties) But, HS interactions do not work, maybe

- No long-range force by HS [Weinberg '64]
- No gravitational coupling for HS [Aragone-Deser '79]

HS interactions DO work in (A)dS

- Gravitational coupling for HS (cubic) [Fradkin-Vasiliev '87]
- Equations for HS (full order) [Vasiliev '90]

Vasiliev's equations

- à la Cartan (ex. Gravity : gauge the isometry algebra)
- HS algebra: HS generalization of the isometry algebra

 $S^{(3)}, S^{(4)}, \ldots$

• "Gauge theory of HS algebra"

Constructive approach (bottom-up)

- \bullet Free theory of HS $$S^{(2)}$$
- Order-by-order construction of interacting vertices
- AdS/CFT and CFT bootstrap program

Constructive approach:

<u>Cubic interactions & HS symmetries</u>

 \checkmark General procedure for interacting actions of HS

Give me field contents,

I will see which (interacting) theories are possible.

- Cubic interactions $S^{(3)}$ (only result)
- \checkmark Deformations of HS symmetries induced by $S^{\scriptscriptstyle (3)}$
- \checkmark HS algebras from $S^{\scriptscriptstyle (3)}$

'HS algebras' associated with classical Lie algebras talk of K. Mkrtchyan (two weeks ago)



I will see which (interacting) theories are possible.



$$S = S^{(2)} + S^{(3)} + S^{(4)} + \cdots$$

 $\delta S = 0 \quad \text{under} \quad \delta \phi = \delta^{\scriptscriptstyle (0)} \phi + \delta^{\scriptscriptstyle (1)} \phi + \delta^{\scriptscriptstyle (2)} \phi + \cdots$

$$\delta^{(0)}S^{(n)} + \dots + \delta^{(n-1)}S^{(2)} = 0$$

solve one by one from n=3





Gauge Algebra

$$\delta_{[\varepsilon_1} \, \delta_{\varepsilon_2]} \phi = \delta_{\llbracket \varepsilon_1, \varepsilon_2 \rrbracket} \phi + \text{(trivial sym.)}$$

field-dependent bracket !

$$\llbracket, \rrbracket = \llbracket, \rrbracket^{(0)} + \llbracket, \rrbracket^{(1)} + \llbracket, \rrbracket^{(2)} + \cdots$$

$$\delta_{[\varepsilon_1}^{(0)} \, \delta_{\varepsilon_2]}^{(1)} \phi = \delta_{[\varepsilon_1, \varepsilon_2]}^{(0)} \phi$$

massless fields of arbi. spin

✓ For given ϕ_1, ϕ_2, ϕ_3 in (A)dS, find all possible $S^{(3)}[\phi_1, \phi_2, \phi_3]$ To Do List

To determine $S^{\scriptscriptstyle (3)}\!,$ we don't need the entire field content, but only the three fields

- ✓ For each of $S^{(3)}[\phi_1, \phi_2, \phi_3]$ find the corresponding deformation of
 - gauge transformation $\delta_{\varepsilon_i}^{(1)}\phi_j$
 - -gauge algebra $[\varepsilon_i, \varepsilon_j]^{(0)}$ i, j = 1, 2, 3
- Explore their consequences (ex. global symmetry)



Technical Difficulties
Handle arbitrary number of indices
(A)dS covariant derivatives



Key Ideas/Technics

 Proper adaptation of ambient-space (or embedding) approach

 $\varphi_{\mu_1\cdots\mu_s}(x) \iff \Phi_{M_1\cdots M_s}(X)$



- Auxiliary variables: similar to string oscillators $\Phi(X,U) \qquad \text{interactions: operators} \propto \partial_{X_i}, \partial_{U_i}$
- Step-by-step construction
 1) Transverse and Traceless part
 2) The rest (unphysical part)

(Old and New) RESULTS

What was known before (incomplete list):

Flat, Metric-like

- All 4d vertices in light-cone [Brink, 2xBengtsson]
- All any d vertices in light-cone [Metsaev]
- Identification of gauge-algebra deformation
 [Bekaert, Boulanger, Lerclerq]
- All vertices in covariant form [Manvelyan, Mkrtchyan, Ruehl; Sagnotti, Taronna]
- All vertices in BRST form [Fotopoulos, Tsulaia; Metsaev]

Strict (A)dS, Frame-like

- All 4d vertices
- All any d non-Abelian vertices

[Fradkin, Vasiliev]

[Vasiliev]

SKIP TECHNICAL DETAILS



CUBIC INTERACTIONS

the most general form up to integration by part and field redefinition

$$S^{(3)}[\Phi_1, \Phi_2, \Phi_3] \stackrel{\text{TT}}{=} \int_{(A)dS} C(Y, Z) \Phi_1(X_1, U_1) \Phi_2(X_2, U_2) \Phi_3(X_3, U_3) \Big|_{\substack{X_i = X \\ U_i = 0}}$$
$$Y_i := \partial_{U_i} \cdot \partial_{X_{i+1}}, \qquad Z_i := \partial_{U_{i+1}} \cdot \partial_{U_{i-1}} \qquad [i \simeq i+3]$$

general solution to the condition $\ \ \delta^{\scriptscriptstyle(0)}S^{\scriptscriptstyle(3)} pprox 0$

$$C(Y,Z) = e^{\lambda \left(Z_1 \partial_{Y_2} \partial_{Y_3} + Z_1 Z_2 \partial_{Y_3} \partial_G + \operatorname{cyc.} + Z_1 Z_2 Z_3 \partial_G^2\right)} K(Y,G) \Big|_{G=Y_i Z_i}$$

• (A)dS effect.
$$\int_{(A)dS} \lambda^{n} I_{\Delta} = c_{\Delta,n} \Lambda^{n} \int_{(A)dS} I_{\Delta}$$

• Coupling Fn.
$$K(Y,G) = \sum_{n=0}^{s_{\min}} k_n \underbrace{Y_1^{s_1-n} Y_2^{s_2-n} Y_3^{s_3-n} G^n}_{\# \text{ of derivatives :}} \\ \underset{s_1+s_2+s_3-2n}{\# s_1+s_2+s_3-2n}$$

Examples

$$\begin{array}{ccc} 2-2-2 & K = k_0 \, Y_1^2 \, Y_2^2 \, Y_3^2 + k_1 \, Y_1 \, Y_2 \, Y_3 \, G + k_2 \, G^2 \\ & & \\ R^{\mu\nu}{}_{\rho\sigma} \, R^{\rho\sigma}{}_{\lambda\kappa} \, R^{\lambda\kappa}{}_{\mu\nu} & & \\ R^{\mu\nu}{}_{\rho\sigma} \, R^{\rho\sigma}{}_{\mu\nu} & & \\ R \end{array} \begin{array}{c} R \\ R \\ & \\ R \end{array}$$

1-1-1
$$K = k_0 Y_1 Y_2 Y_3 + k_1 G$$
$$tr[F^{\mu}{}_{\nu} F^{\nu}{}_{\rho} F^{\rho}{}_{\mu}] tr[F^{\mu}{}_{\nu} F^{\nu}{}_{\mu}]$$

$$2-1-1 K = k_0 Y_1^2 Y_2 Y_3 + k_1 Y_1 G$$

$$R^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma}$$
Gravitational
interaction

CUBIC INTERACTIONS

the most general form up to integration by part and field redefinition

$$S^{(3)}[\Phi_1, \Phi_2, \Phi_3] \stackrel{\text{TT}}{=} \int_{(A)dS} C(Y, Z) \Phi_1(X_1, U_1) \Phi_2(X_2, U_2) \Phi_3(X_3, U_3) \Big|_{\substack{X_i = X \\ U_i = 0}}$$

$$Y_i := \partial_{U_i} \cdot \partial_{X_{i+1}}, \qquad Z_i := \partial_{U_{i+1}} \cdot \partial_{U_{i-1}} \qquad [i \simeq i+3]$$

general solution to the condition $\delta^{(0)}S^{(3)} \approx 0$

$$C(Y,Z) = e^{\lambda \left(Z_1 \partial_{Y_2} \partial_{Y_3} + Z_1 Z_2 \partial_{Y_3} \partial_G + \operatorname{cyc.} + Z_1 Z_2 Z_3 \partial_G^2\right)} K(Y,G) \Big|_{G=Y_i Z_i}$$

• Coupling Fn.
$$K(Y,G) = \sum_{n=0}^{s_{\min}} k_n Y_1^{s_1-n} Y_2^{s_2-n} Y_3^{s_3-n} G^n$$

DEFORMATIONS

gauge transformation deformations $\delta^{(1)}$

 $\delta_{E_1}^{(1)} \Phi_3(X, U) \stackrel{\text{TT}}{=} -\frac{1}{2} \prod_{\Phi} \partial_{Y_1} \bar{C}(Y, Z) E_1(X_1, U_1) \Phi_2(X_2, U_2) \Big|_{X_i = X_i = X_i = 0}$ + field redefinition !

gauge algebra (bracket) deformations $[\ , \]^{(0)}$ $[\ E_1 \ , E_2 \]^{(0)} \stackrel{\text{TT}}{=} \frac{1}{4} \prod_E (\partial_{Y_1} \partial_{Z_1} + \partial_{Y_2} \partial_{Z_2}) \overline{C}(Y, Z) E_1 E_2 |$ + parameter redefinition !

$$\bar{C}(Y,Z) = e^{\lambda \left(Z_1 \partial_{Y_2} + Z_2 \partial_{Y_1} + Z_1 Z_2 \partial_G\right) \partial_{Y_3}} K(Y,G) \Big|_{G=Y_i Z_i}$$

The redefinitions exists such that $\delta_{E_i}^{(1)} = 0$ and/or $[\![E_i, E_j]\!]^{(0)} = 0$?

RESULT : CLASSIFICATION OF CUBIC INTERACTIONS

• Four different classes of couplings for a given $s_1 - s_2 - s_3$ interaction

 Depending on spins, certain classes can be empty

	$\#_{\partial}$	$\delta_{E_1}^{(1)}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$
	$s_1 + s_2 + s_3$	= 0	= 0	= 0
Class I	÷	÷	•	:
	$2s_1$	= 0	:	:
	:	$\neq 0$	•	÷
Class II	÷	÷	•	÷
	$2 s_2$:	=0	=0
Class III	÷	÷	$\neq 0$	Λ
	÷	÷	•	÷
	$2s_3$:		Λ
Class IV	÷	•	• •	$\neq 0$
	:	:	•	•
	$s_1 + s_2 - s_3$	$\neq 0$	$\neq 0$	$\neq 0$

 $s_1 \ge s_2 \ge s_3$

DEFORMATIONS

CLASS I

- Non-deforming $\, \delta^{\scriptscriptstyle (1)}_{\scriptscriptstyle E_i} = 0 \,$ Abelian couplings
- Expressible in terms of any two curvatures
 - \bullet R_1 and R_2 \bullet R_1 and R_3 \bullet R_2 and R_3

CLASS IV

• Non-Abelian couplings

$$\delta_{E_i}^{(1)} \neq 0$$
$$\llbracket E_i, E_j \rrbracket^{(0)} \neq 0$$

	<i></i>	s (1)	s (1)	c (1)
	$\#\partial$	O_{E_1}	0_{E_2}	0_{E_3}
Class I	$s_1 + s_2 + s_3$	= 0	= 0	= 0
	÷	÷	÷	÷
	$2 s_1$	= 0	÷	÷
Class II	:	$\neq 0$	÷	÷
	÷	÷	÷	÷
	$2 s_2$	÷	= 0	= 0
Class III	÷	÷	$\neq 0$	Λ
	÷	÷	÷	÷
	$2s_3$	÷	÷	Λ
	÷	:	:	$\neq 0$
Class IV	÷	÷	÷	÷
		10	10	10

	#∂	$\delta^{(1)}_{E_1}$	$\delta^{(1)}_{E_2}$	$\delta^{(1)}_{E_3}$
	$s_1 + s_2 + s_3$	= 0	= 0	= 0
Class I	÷	÷	÷	÷
	$2 s_1$	= 0	÷	÷
	:	$\neq 0$	÷	÷
Class II	÷	÷	÷	÷
	$2 s_2$	÷	= 0	= 0
	:	÷	$\neq 0$	Λ
Class III	:	÷	÷	÷
	$2s_3$	÷	÷	Λ
	:	:	:	$\neq 0$
Class IV	÷	÷	÷	÷
	$s_1 + s_2 - s_3$	$\neq 0$	$\neq 0$	$\neq 0$
0			> (<u> </u>
s_1	$\leq S_2$	$2 \leq$	<u> </u>	>3



• Deforming $\, \delta_{\scriptscriptstyle E_1}^{\scriptscriptstyle (1)} eq 0$, but Abelian couplings

- Expressible in terms of the curvatures R₂ and R₃
 Generalized Bel-Robinson currents
- Fields 2 and 3 are charged w.r.t field 1
 - HS algebra multiplet

CLASS II

Class I Class II	$ \begin{array}{c} \#_{\partial} \\ s_1 + s_2 + s_3 \\ \vdots \\ \hline \\ \frac{2 s_1}{\vdots} \\ \vdots \\ \vdots \end{array} $	$\delta_{E_1}^{(1)} = 0$ \vdots $= 0$ $\neq 0$	$ \begin{array}{c} \delta_{E_2}^{(1)} \\ = 0 \\ \vdots \\ \vdots \\ \vdots \end{array} $	$ \begin{array}{c} \delta_{E_3}^{(1)} \\ = 0 \\ \vdots \\ \vdots \\ \vdots \end{array} $
Class I Class II	$s_1 + s_2 + s_3$ \vdots $-2 s_1$ \vdots \vdots	$= 0$ $= 0$ $\neq 0$	= 0 : : :	= 0 : :
Class I Class II	$\frac{2 s_1}{\frac{1}{2} s_1}$	$ \vdots \\ = 0 \\ \neq 0 $		
Class II	$2 s_1$:	$= 0$ $\neq 0$:	:
Class II	:	$\neq 0$	÷	
Class II	:			:
		:	÷	÷
	$2 s_2$	÷	= 0	= 0
	:	÷	$\neq 0$	Λ
Class III	÷	:	÷	÷
	$2s_{3}$	÷	÷	Λ
	÷	:	÷	$\neq 0$
Class IV	÷	÷	÷	÷
	$s_1 + s_2 - s_3$	$\neq 0$	$\neq 0$	$\neq 0$

CLASS III

• Non-Abelian couplings with

$$\delta_{E_1}^{(1)} \neq 0 \qquad \qquad \delta_{E_2}^{(1)} \neq 0$$

$$\llbracket E_1, E_2 \rrbracket^{(0)} \neq 0$$

$$\delta_{E_3}^{(1)} = \mathcal{O}(\Lambda)$$
$$\llbracket E_1, E_3 \rrbracket^{(0)} = \mathcal{O}(\Lambda)$$
$$\llbracket E_2, E_3 \rrbracket^{(0)} = \mathcal{O}(\Lambda)$$

Qualitative difference between Flat Space & (A)dS

- Gravitational couplings
 - Aragone-Deser No-go
 - Fradkin-Vasiliev interactions

s-s-2	$\#_\partial$	$\delta^{\scriptscriptstyle (1)}_{\scriptscriptstyle E_1}$	$\delta^{(1)}_{\scriptscriptstyle E_2}$	$\delta^{\scriptscriptstyle (1)}_{\scriptscriptstyle E_3}$
	2s + 2	0	0	0
Class I	2s	0	0	0
Class III	2s - 2	*	*	Λ

$$\delta_{E_3}^{(1)} \Phi = \left(\frac{\partial^{2s-3} + \Lambda \partial^{2s-5} + \dots + \Lambda^{s-2} \partial}{\mathbf{field redefinition}} \right) E_3 \Phi$$

field redefinition
General coordinate transf.

- YM-like couplings
 - colored gravity in (A)dS !

s - s - 1	$\#_{\partial}$	$\delta^{\scriptscriptstyle (1)}_{\scriptscriptstyle E_1}$	$\delta^{(1)}_{\scriptscriptstyle E_2}$	$\delta^{\scriptscriptstyle (1)}_{\scriptscriptstyle E_3}$
Class I	2s + 1	0	0	0
Class III	2s - 1	*	*	Λ

f.

$$\delta_{E_3}^{(1)} \Phi = \underbrace{\partial^{2s-2} + \Lambda \partial^{2s-4} + \dots + \Lambda^{s-1}}_{\text{field redefinition}} E_3 \Phi$$



Next step ?

Consistency of Algebra ⇒ Global symmetry

• Quartic condition : $\delta^{(0)}S^{(4)} + \delta^{(1)}S^{(3)} + \delta^{(2)}S^{(2)} = 0$ (work in progress)

<u>GLOBAL HIGHER SPIN SYMMETRIES</u> (HIGHER SPIN ALGEBRAS)

all bracket structures!

Killing Equations for (A)dS background

 $\partial_{(M_1} E_{M_2 \cdots M_s)} = 0$

Lie Algebra with bracket [,]⁽⁰⁾

Stacket entirely determined by TT alone

 ★ Must satisfy Jacobi identity
 ⇒ Determine HS algebra and non-Abelian cubic interactions

* Must be consistent with $\delta_{[E_1}^{(1)} \delta_{E_2]}^{(1)} = \delta_{[E_1, E_2]}^{(1)}$ \Rightarrow Determine **representation** and **Abelian** (class II) cubic interactions

Jacobi identity

- ✓ Lie Algebra Bracket $[E_i, E_j]_k$ from $S^{(3)}[\Phi_i, \Phi_j, \Phi_k]$
 - Independent of spectrum (the entire field contents)
- ✓ Jacobi identity
 - Spectrum matters
- $\sum_{\boldsymbol{\ell}} \left[\left[\left[E_{[i}, E_{j} \right] \right]_{\boldsymbol{\ell}}, E_{k]} \right] \right]_{\boldsymbol{m}} = 0$

- Vasiliev spectrum: single field for each spin
 - Uniqueness (Vasiliev algebra)

[Boulanger, Ponomarev, Skvortsov, Taronna]

• Explicit structures

[EJ and K. Mkrtchyan]

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✓ Two more examples of Jacobi

ex.1 HS algebra in flat-space ?

- Consider spin 2 interacting with HS particles From our analysis, $[T_2, T_s] = 0$ "spin 2 cannot be graviton"
- Is it possible that **spin 2** interacts with **HS** through **non-Abelian** interactions ?

• Jacobi imposes $[T_2, T_2] = 0$

No "HS algebra" containing Poincaré as sub-algebra

This is **Coleman-Mandula** thm. but **without** any assumption on the **spectrum** !

ex.2 Tensionless limit of ST around (A)dS ?

 Cubic interactions of massive HS fields in 1st Regge traj. open bosonic string theory

$$C \sim \frac{1}{g_o} e^{Y_1 + Y_2 + Y_3 + \frac{1}{\alpha'} (Z_1 + Z_2 + Z_3)}$$

• Cubic interactions of massless HS fields in (A)dS with the choice of coupling fn $K(Y,G) = \frac{1}{a}e^{Y_1+Y_2+Y_3}$

$$C = \frac{1}{g} e^{Y_1 + Y_2 + Y_3 + \lambda(Z_1 + Z_2 + Z_3)}$$

 $\begin{aligned} & \mathsf{Gauge inv. cubic interactions} \\ & C(Y,Z) = e^{\lambda \left(Z_1 \partial_{Y_2} \partial_{Y_3} + Z_1 Z_2 \partial_{Y_3} \partial_G + \operatorname{cyc.} + Z_1 Z_2 Z_3 \partial_G^2\right)} K(Y,G) \Big|_{G=Y_i Z_i} \\ & \mathsf{Generic cubic interactions} \\ & S^{(3)}[\Phi_1, \Phi_2, \Phi_3] \stackrel{\text{TT}}{=} \int_{(A)dS} C(Y,Z) \, \Phi_1(X_1, U_1) \, \Phi_2(X_2, U_2) \, \Phi_3(X_3, U_3) \Big|_{X_i=X} \\ & Y_i := \partial_{U_i} \cdot \partial_{X_{i+1}}, \qquad Z_i := \partial_{U_{i+1}} \cdot \partial_{U_{i-1}} \quad [i \simeq i+3] \end{aligned}$

ex.2 Tensionless limit of ST around (A)dS ?

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• Cubic interactions of massless HS fields in (A)dS with the choice of coupling fn $K(Y,G) = \frac{1}{q} e^{Y_1 + Y_2 + Y_3}$

$$C = \frac{1}{g} e^{Y_1 + Y_2 + Y_3 + \lambda(Z_1 + Z_2 + Z_3)}$$

- gives HS algebra given by Moyal product (so satisfy Jacobi)
- in the condition that the HS fields are reducible !

Reducible HS spectrum do appear in the BRST formulation of a tensionless limit of ST

Summary

- All cubic interactions of massive and (partially-)massless HS fields
- Full classification of massless interactions
- All bracket structures of HS algebra

Outlook

- Higher order interactions (work in progress)
- AdS/CFT computations, CFT bootstrap program
- Generalization to fermions and mixed-sym. fields

THANK YOU