# A Review of Higher Spin Field Theory

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# Higher Spins

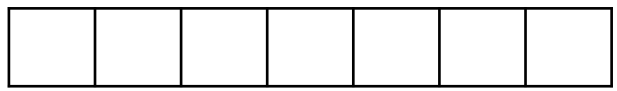
- Hadronic excitations
- Higher excitations in String Theory

**Can a better field-theoretical understanding of higher spins give useful lessons to String (Field) Theory?**

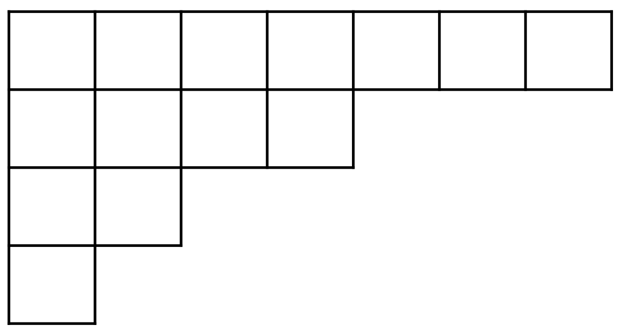
# Massive Higher Spins

Around **flat d** dimensions, massive little group: SO(d-1)

In d=4, only **symmetric** YD rep



In d>4, various **mixed-symmetry** YD rep



Let us focus first on symmetric **massive** **Higher Spin** fields

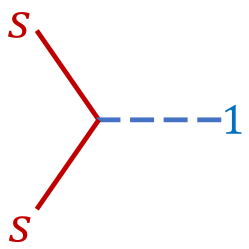
❖ Free Lagrangian by Singh Hagen in ’74

❖ Consistent Interactions?

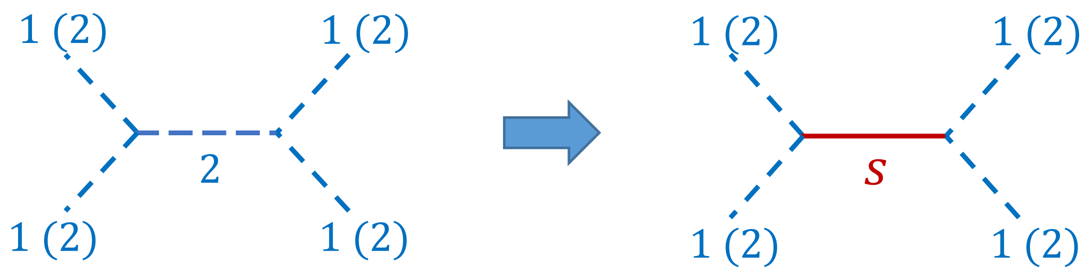
- Minimal EM interaction to charged massive HS

  ✓ Required non-minimal $F_{\mu\nu}$ interaction → Causality Problem

  ✓ String theory gives higher $F^n$ interactions




- Causality of $F_{\mu\nu}$ or $R_{\mu\nu\varrho\sigma}$ interactions requires <u>infinitely many</u> **massive HS**




☆ **Lesson**: HS interactions ↔ Higher derivatives (<u>dim.ful parameter</u>)

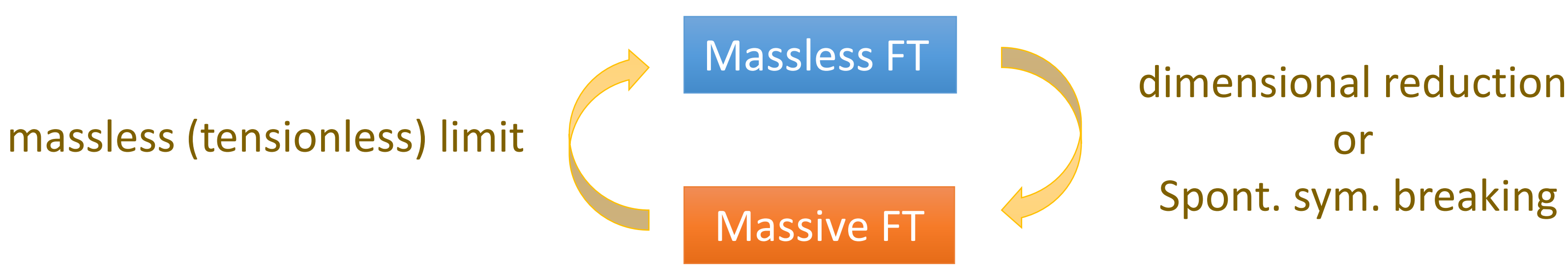
Other consistency of **massive** interactions?

Even, classical consistency of DoF is not obvious

- Simplest example of "higher spin" → spin two
  - Massive spin-two interaction problem → Massive Gravity
  - Consistent massive gravity potential term is very restrictive,

    but it turned out to be natural ones from massless gravity viewpoint

- Massless theory may give a hint



Basic features and Novelties of massless higher spin dynamics

# Massless Higher Spins
## and
# Their Interactions

# Massless (symmetric) Higher Spins

- Free Lagrangian by Fronsdal '78 (via massless limit of massive HS)

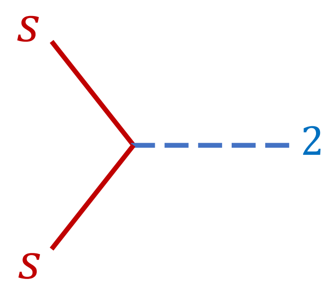
$$S_{\text{Fronsdal}} = \int d^d x\, \varphi^{\mu_1 \cdots \mu_s} \left( \Box + \cdots \right) \varphi_{\mu_1 \cdots \mu_s}$$

  - Gauge Symmetry: $\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \varepsilon_{\mu_2 \cdots \mu_s)}$
  - Subtlety of Trace Constraints
    - Equivalent formulation w/o trace constraints (inspired by SFT)

- Consistent **Interactions**?
  - Various **problems** such as Weinberg '64 (No long range interaction of HS)
  - **Gauge Invariance** (with a nonlinear deformation)

Gravitational minimal interaction of massless spin s

→ Fronsdal Lagrangian w/ covariant derivatives: $\mathcal{L}_{\text{Fronsdal}}(\varphi, \nabla\varphi)$




✓ Manifestly invariant under diffeomorphism

✓ Invariance under HS gauge transform?

- In principle, terms from $[\nabla_\mu, \nabla_\nu] \sim R_{\mu\nu\rho\sigma}$
- $s=1$ : no such term → Spin 1 is 'matter' w.r.t Gravity
- $s=3/2$ : term prop to $R_{\mu\nu}$ → Compensate by $\delta(\sqrt{g}\,R)$ with $\delta g_{\mu\nu}$

➡ Spin 3/2 and 2 in a SUSY multiplet

- $s>2$ : term prop to $R_{\mu\nu\varrho\sigma}$ → Impossible to save HS gauge sym

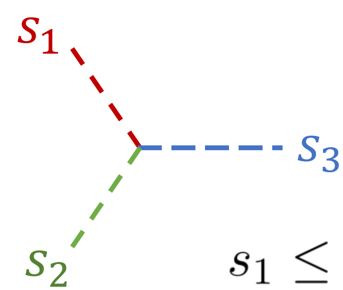
**Incompatibility between Diffeomorphism and HS gauge symmetry**

## ❖ Construction of gauge invariant interaction vertices

- Perturbative expansion

$$S = S_0 + S_1 + \cdots$$

$$\delta \varphi = \partial \varepsilon + T_1(\varphi, \varepsilon) + \cdots$$




$$s_1 \leq s_2 \leq s_3$$

- Gauge invariant cubic vertices

$$S_1 = \sum_{n=0}^{s_1} g_{s_1+s_2+s_3-2n} \, V_{s_1+s_2+s_3-2n}$$

# of $\partial$

Diffeomorphism

2-2-2 $V_2 \sim R$ $\quad V_4 \sim R^{\mu\nu}{}_{\rho\sigma} \, R^{\rho\sigma}{}_{\mu\nu}$ $\quad V_6 \sim R^{\mu\nu}{}_{\rho\sigma} \, R^{\rho\sigma}{}_{\alpha\beta} \, R^{\alpha\beta}{}_{\mu\nu}$

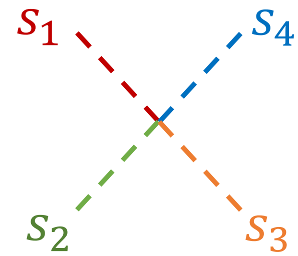
0-0-2 $V_2 \sim (\nabla\phi)^2$ General covariance of spin 0 and 1

1-1-2 $V_2 \sim g^{\mu\rho} \, g^{\nu\sigma} \, F_{\mu\nu} \, F_{\rho\sigma}$ $\quad V_4 \sim R^{\mu\rho\nu\sigma} \, F_{\mu\nu} \, F_{\rho\sigma}$

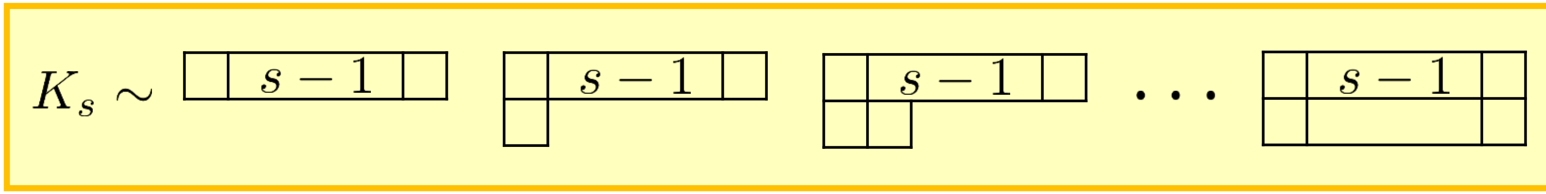
$s$-$s$-2 $V_{2s-2}$ $\quad V_{2s}$ $\quad V_{2s+2}$ Loose general covariance

➡ Consistent cubic interactions are higher derivative types

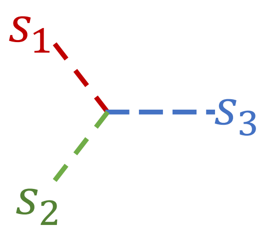
- Consistent quartic vertices: seemingly, nothing local




  - One way to see this → Global Sym
    - ✓ Generator fixed by Killing eq, $\partial\varepsilon = 0$

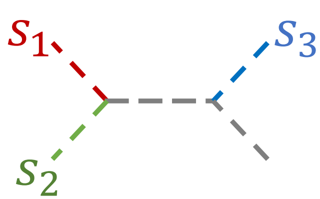



    - ✓ Bracket fixed by cubic vertices




$$[K_{s_1}, K_{s_2}] = (\cdots)\, K_{s_3}$$

    - ✓ Quartic consistency → Jacobi idenitity




$$[[K_{[s_1}, K_{s_2}], K_{s_3]}] = 0$$

**Impossible →** No massless HS interactions in **FLAT SPACE**

Remind

★ **Lesson**: HS interactions ↔ Higher derivatives (dim.ful parameter)

➡ Massive HS and/or Massless HS in **(A)dS**

Mass and Cosmological Const. play somewhat similar role

❖ Gauge invariant cubic vertices in (A)dS

$$S_1 = \sum_{n=0}^{s_1} g_{s_1+s_2+s_3-2n}\, V_{s_1+s_2+s_3-2n}$$

**Max # of ∇**

- Very analogous to flat space case
- Important **difference**:

**General covariance of spin s !!**

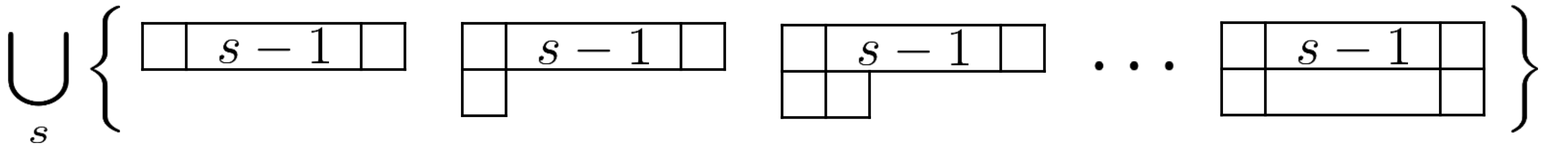
$s$-$s$-2 $\quad V_{2s-2} \quad V_{2s} \quad V_{2s+2}$

$$\sim (\nabla\varphi_s)^2 + \frac{W}{\Lambda}(\varphi_s\nabla^2\varphi_s) + \cdots + \frac{W}{\Lambda^{s-2}}(\varphi_s\nabla^{2s-4}\varphi_s)$$

➡ Different spins in a multiplet of **HS Algebra**

# Higher Spin Algebra

- What is it? : Lie algebra generated by




- Does this sym exists? If not, the quartic consistency would fail even in (A)dS

- **Vasiliev's HS Algebra** '87
  - Various Equivalent Definitions
    - Star product algebra in a certain oscillator space
    - Maximal quotient of UEA of so(2,d) (relation to min orbit)
    - Maximal symmetry of free **conformal scalar** in d-1 dim!
  - Contains all even (and odd) spin generators
  - Flato-Fronsdal

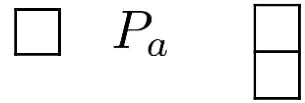
$$\text{Rac} \otimes_{(\text{sym})} \text{Rac} = \bigoplus_{\text{even } s, (\text{odd } s)} D(s+d-2, s)$$

massless spin s rep

# Higher Spin Gravity

# $1^{st}$ order formulation of Higher Spins

❖ $1^{st}$ order formulation of **Gravity**

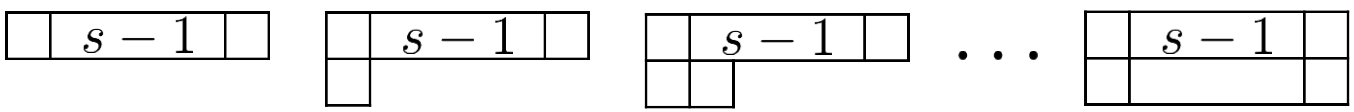


$P_a$ $M_{ab}$

| | | |
|---|---|---|
| 1-form connection | $e^a$ | $\omega^{ab}$ |
| 2-form curvature | $T^a$ | $R^{ab}$ |

=0 → fix

❖ $1^{st}$ order formulation of **massless spin s**




| | | | | | |
|---|---|---|---|---|---|
| 1-form | $A^{a_1\cdots a_{s-1}}$ | $A^{a_1\cdots a_{s-1},b}$ | $A^{a_1\cdots a_{s-1},b_1b_2}$ | | $A^{a_1\cdots a_{s-1},b_1\cdots b_{s-1}}$ |
| 2-form | $F^{a_1\cdots a_{s-1}}$ | $F^{a_1\cdots a_{s-1},b}$ | $F^{a_1\cdots a_{s-1},b_1b_2}$ | | $F^{a_1\cdots a_{s-1},b_1\cdots b_{s-1}}$ |

=0 → fix =0 → fix =0 → ... → fix

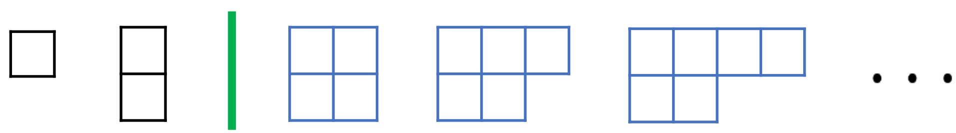
- This step also gives EoM → difficult to disentangle **EoM** and **Constraints**
- Fradkin Vasiliev construction $$S=\int\sum_{r=0}^{s-1}\frac{a_r}{\Lambda^r}\,(F^{(s-1,r)})^2$$

✓ Inconsistent for **Quartic Order**

# Unfolded Equations

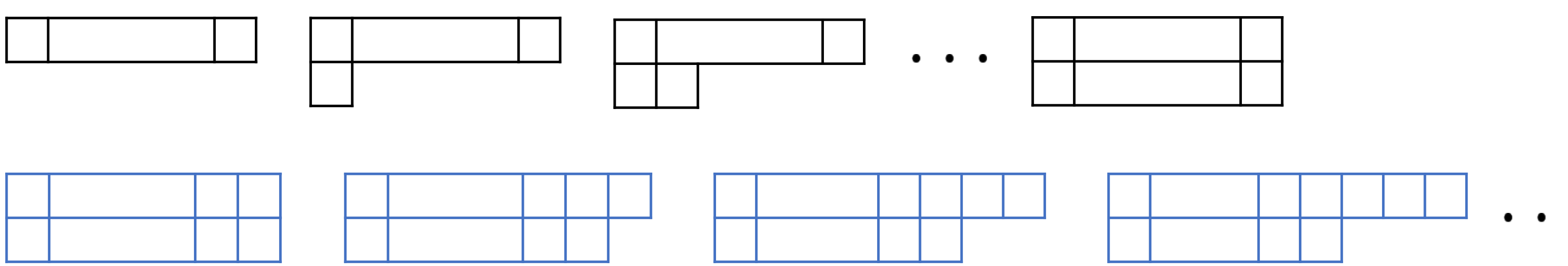
- Universal treatment of **EoM** & **Constraints**
- No privilege to metric
- Gravity ex.

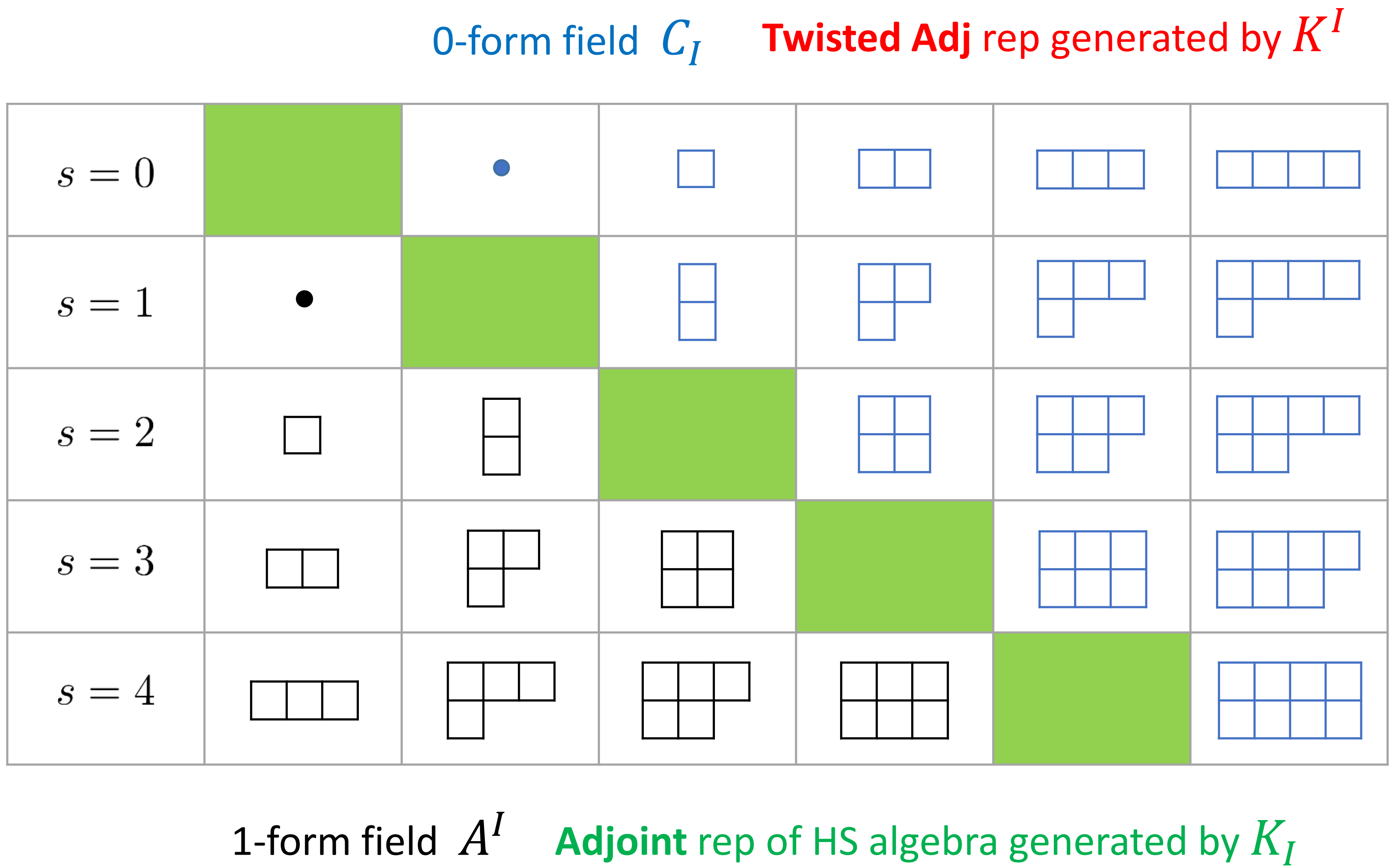
| | | | | |
|---|---|---|---|---|
| Fields: | $e^a$ | $\omega^{ab}$ | $C^{ab,cd}$ | $\cdots$ |
| Eqns: | $De^a = 0$ | $D\omega^{ab} = e_c\, e_d\, C^{ac,bd}$ | $D\,C^{ab,cd} = e_e\, C^{abe,cd} + \cdots$ | |



**1-form** **0-form**

- Spin s




0-form field $C_I$
Twisted Adj rep generated by $K^I$
$s = 0$
$s = 1$
$s = 2$
$s = 3$
$s = 4$
1-form field $A^I$
Adjoint rep of HS algebra generated by $K_I$

- Ansatz for nonlinear HS equations

$$d\,A^I + f^I_{JK}(C)\,A^J\,A^K = 0$$
$$d\,C_I + g^J_{IK}(C)\,C_J\,A^K = 0$$

- Frobenius condition → impose conditions on $f^I_{JK}(C)$ and $g^J_{IK}(C)$
  - Free Differential Algebra (FDA)
  - Infinite dimensional Lie Algebroid
- Vasiliev identified on $f^I_{JK}(C)$ and $g^J_{IK}(C)$ up to $O(C^3)$ ['88, '89]

- "**Vasiliev's Equations**" ['90]
  - Similar eqns which **GENERATE** $f^I_{JK}(C)$ and $g^J_{IK}(C)$
  - Key ideas: extend ("double") the fiber, s.t "$f^I_{JK}$ and $g^J_{IK}$" become cnst
    but, fields are subjects to algebraic constraints

# Vasiliev's Equation in 4d

- HS algebra realized by oscillators $Y_A Y_B$ , $Y_A Y_B Y_C Y_D$ , …

$$A^I \quad \rightarrow \quad A(Y) \qquad\qquad C_I \quad \rightarrow \quad C(Y)$$

- Doubling of oscillator space: $A(Y,Z)\,, \quad C(Y,Z)\,, \quad S_A(Y,Z)$

  new fields

- The Equations

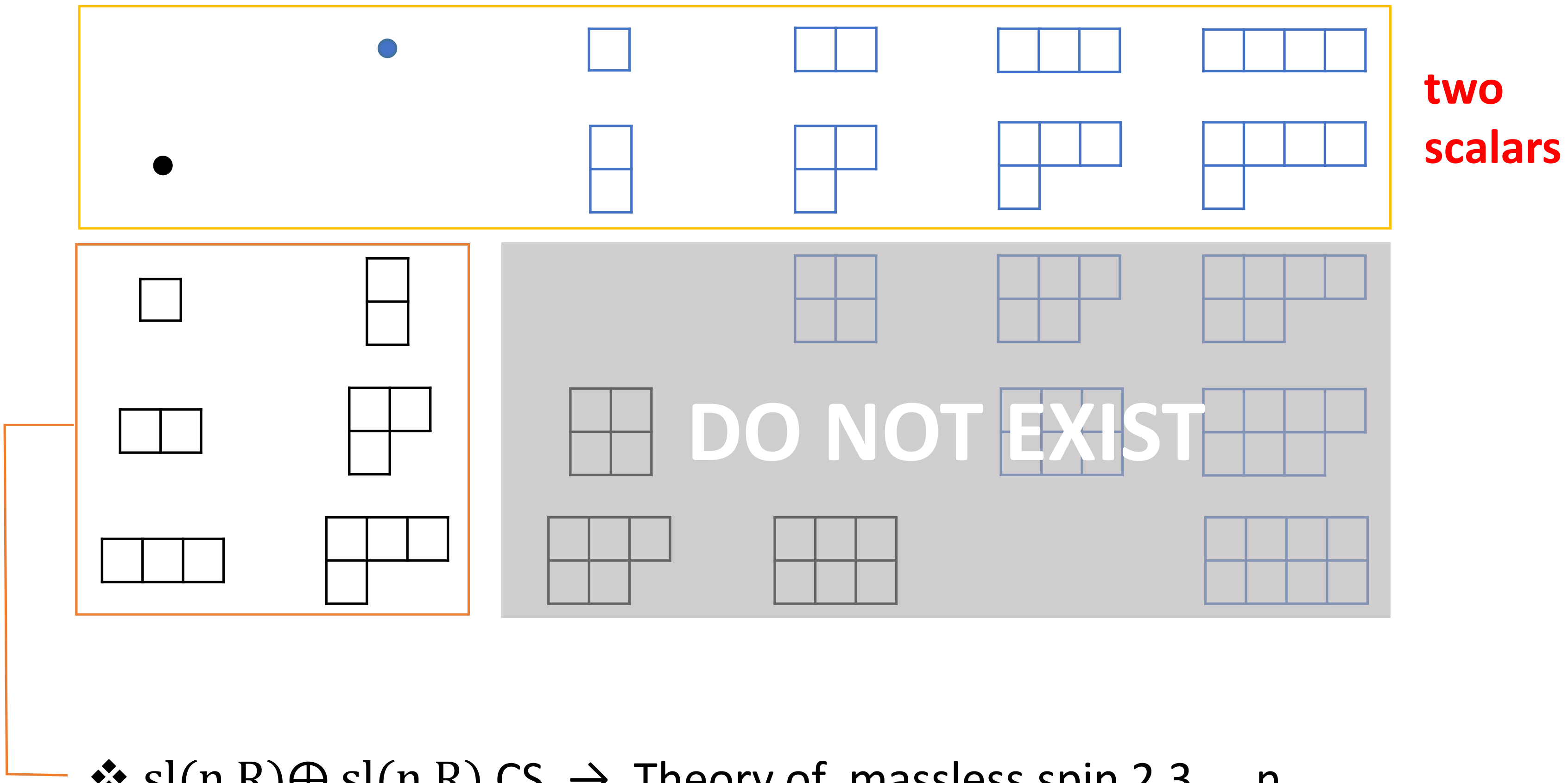
$$d\,A + A \star A = 0 \qquad d\,C + [A \stackrel{\star}{,} C] = 0 \qquad d\,S_\alpha + [A \stackrel{\star}{,} S_\alpha] = 0$$

Algebraic constraints

$$[C \stackrel{\star}{,} S_\alpha] = 0 \qquad [S_\alpha \stackrel{\star}{,} S_\beta] = \epsilon_{\alpha\beta}(1 + e_\star^{i\,\Theta_\star(C)} \star C)$$

- Interaction Ambiguity $\Theta_\star(C) = \theta_0 + \theta_2\, C^{\star 2} + \cdots$
  - Parity invariance $\rightarrow$ $\theta_0 = 0$ or $\pi$ , $\theta_{n>0} = 0$
- A few exact solutions
- No action principle yet

# Higher Spin Theories in 3d



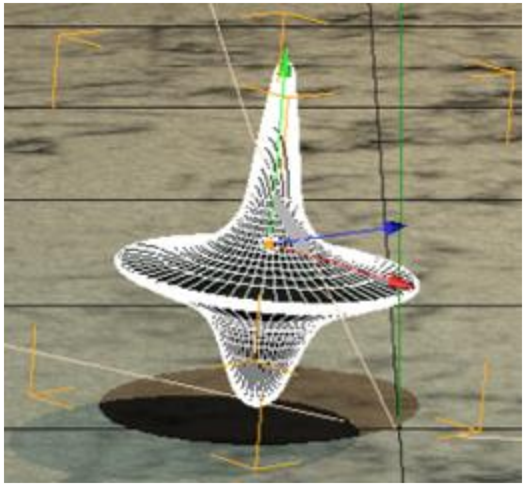

❖ sl(n,R)⊕ sl(n,R) CS → Theory of massless spin 2,3,...,n

❖ 3d Vasiliev Eq → Theory of massless spin 2,3,...,∞ and **two scalars**

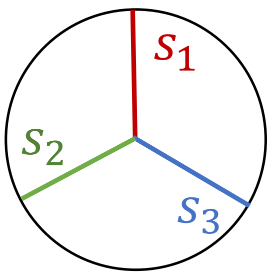
Can be viewed as CS gauge sector coupled to matter sector

AdS/CFT conjectures
Involving higher spin gravity

# Higher Spin Holography



| $AdS_{d+1}$ | $CFT_d$ |
|---|---|
| Field contents | Single trace operators |
| • Fields of mass M & spin s | • Operators of dimension Δ & spin s |
| • Massless spin s fields | • Spin s conserved current operators |
| Cubic Interactions | 3pt functions |




$$\langle \mathcal{O}_{s_1}(x_1)\,\mathcal{O}_{s_2}(x_2)\,\mathcal{O}_{s_3}(x_3)\rangle$$

Classifications match

- Holography for HS gravity in $AdS_{d+1}$ with $d \geq 3$
  - HS sym: max sym of conf scalar ($CFT_3$ with HS sym → only free scalar/spinor)
  - Flato-Fronsdal: operators **bilinear** in $\phi$ → Conserved currents of any spins

**Vector Model**

- ❖ $AdS_{d+1}/CFT_d$ $(d \geq 3)$
  - U(N)/O(N) Scalar Vector Model ⇔ (Non)-minimal Vasiliev Theory

- ❖ $AdS_4/CFT_3$
  - Spinor Vector Model ⇔ Vasiliev Theory with $\theta_0 = \pi$
    - ✓ Test for a large class of 3pt functions
  - Critical Models ⇔ AdS scalar with different BC
  - Parity violating Vasiliev Theory with $\theta_0$ ⇔ CS coupling to 3d CFT
    - ✓ Test for a few 3pt fns, but not conclusive
  - Open question: Vasiliev theory with other $\theta_n$ ?

## ❖ $AdS_3/CFT_2$

- Background of 3d Vasiliev theory, parametrized by $\lambda$
- HS sym: $hs(\lambda) = UEA(sl_2)/C_2(\lambda)$
- Asymptotic symmetry: $W_\infty(\lambda)$
  - Nonlinear sym (not a Lie algebra)
  - Does not contain $hs(\lambda)$
- Duality: Vasiliev theory ⇔ $W_N$ minimal model CFT

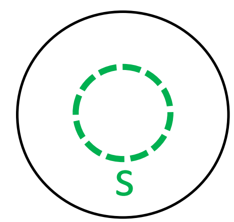
  ✓ Test for spectrum and a few 3pt functions
- BH-like solutions in HS CS theory

# 1 Loop Test for HS AdS/free CFT

❖ Dictionary: 1 Loop in AdS ⇔ 1/N in CFT

| Free CFT | HS Gravity in AdS |
|---|---|
| No 1/N Correction | No Loop Correction? |

❖ Test for **Vacuum Energy**




$= E_s$

- Total vacuum energy: sum of ∞ contributions
  - For non-minimal model: $\sum_{all\ s} E_s = 0$
  - For minimal model: $\sum_{even\ s} E_s = E_{bd\ scalar}$

    ➡ Shift of HS Coupling Const. N→N-1
- 5d HS Gravity dual to free spin 1 in 4d : N→N-2

# Comments on
# Other Topics / Recent Progress

# Other Topics

- Other Formulation
  - BRST related
  - Tensorial space
  - Other metric-like form.
  - World-Line formalism
- Other Spectra
  - Mixed Sym HS
  - Partially Massless and Massive HS
  - Conformal HS
  - Non-Relativistic HS
- Extensions of Vasiliev's Eq
  - SUSY
  - Color Decoration
  - Higher Form
- Relation to String Theory
  - Tensionless Limit
  - WS proposal

# Recent Progress

- Better understanding of Vasiliev's Equation
  - Explicit derivation of cubic vertices
  - Holographic identification of one nonlocal quartic vertex

- Generalizations
  - Extension of HS Algebras to Multi-Ptcl. & Partially Massless ones
  - Various Properties of Conformal HS
  - Holography for Stringy Extensions
  - Rainbow Vacua of Colored (HS) Gravity

Thank You for the Attention