Recent Progresses in
Higher Spin Gravity

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An overview of "old things"

❖ Set of equations for free spin s [Fierz-Pauli 39, …]

❖ Lagrangian for free spin s [Singh-Hagen 74, Fronsdal 78, …]

❖ Interaction problems

- No-go for massless HS [Weinberg 64, Coleman-Mandula 67, …]

- A few ex. of cubic interactions [Berends-Burgers-van Dam 84, …]

- Light-cone approach for cubic interactions (and quartic)
  [Bengtsson^2-Brink 83, Fradkin-Metsaev 91, …]

- Frame-like approach [Fradkin-Vasiliev 87,…]

  ✓ Unfolding [Vasiliev, …]

  ✓ "Vasiliev equation" [Vasiliev 90, …]

An overview of "old things"

❖ New Activities

- Vasiliev equations [Sezgin, Sundell et al]

- HS aspects of AdS/CFT [Klebanov-Polyakov, Sezgin-Sundell, …]

- New formulations of free HS and cubic interactions […]

- Various Generalizations […]

    ▪ Conformal HS Gravity [Fradkin-Linetsky 89, … , Segal 04, …]

❖ Higher spin holography

- AdS4/CFT3 [Giombi-Yin 10, …]

- AdS3/CFT2 […, Gaberdiel-Gopakumar 11, …]

Massless (symmetric) Higher Spins

❖ Free Lagrangian by Fronsdal '78 (via massless limit of massive HS)

$$S_{\text{Fronsdal}} = \int d^d x \, \varphi^{\mu_1 \cdots \mu_s} \left(\Box + \cdots \right) \varphi_{\mu_1 \cdots \mu_s}$$

▪ Gauge Symmetry: $\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \varepsilon_{\mu_2 \cdots \mu_s)}$

▪ Global symmetry:

   ✓ Generator fixed by Killing eq, $\partial \varepsilon = 0$

$$K_s \sim \boxed{\begin{array}{|c|}\hline s-1 \\\hline\end{array}} \quad \begin{array}{c} s-1 \\ \end{array} \quad \begin{array}{c} s-1 \\ \end{array} \quad \cdots \quad \begin{array}{c} s-1 \\ \end{array}$$

   ✓ Bracket fixed by cubic vertices

   ✓ Jacobi id. **cannot** hold in **FLAT SPACE d>4**

   → Gauge invariance breaks down at quartic interactions

❖ Higher Spin in (A)dS

- Jacobi id. **does** hold → HS Algebra

- Contains all even (and odd) spin generators

$$\bigcup_{s} \left\{ \boxed{\;s-1\;} \quad \boxed{\;s-1\;} \quad \boxed{\;s-1\;} \quad \cdots \quad \boxed{\;s-1\;} \right\}$$

- Various Equivalent Definitions of HS Algebra

  ➢ Star product algebra in a certain oscillator space

  ➢ Maximal quotient of UEA of so(2,d) (Joseph ideal)

  ➢ Deformation quantization of minimal coadjoint orbit

  ➢ Maximal symmetry of free **conformal scalar** in d-1 dim!

- Flato-Fronsdal

$$\mathrm{Rac} \otimes_{(\mathrm{sym})} \mathrm{Rac} = \bigoplus_{\mathrm{even}\, s,(\mathrm{odd}\, s)} D(s+d-2, s)$$

massless spin s rep

1st order formulation of Higher Spins

❖ 1st order formulation of **Gravity**

$$\square \quad P_a \qquad \begin{array}{c}\square \\ \square\end{array} \quad M_{ab}$$

1-form connection $\qquad e^a \qquad \omega^{ab}$

2-form curvature $\qquad T^a \; \text{=0 → fix} \qquad R^{ab}$

❖ 1st order formulation of **massless spin s**

1-form $\quad A^{a_1 \cdots a_{s-1}} \quad A^{a_1 \cdots a_{s-1},b} \quad A^{a_1 \cdots a_{s-1},b_1 b_2} \quad \cdots \quad A^{a_1 \cdots a_{s-1},b_1 \cdots b_{s-1}}$

2-form $\quad F^{a_1 \cdots a_{s-1}} \text{=0 → fix} \; F^{a_1 \cdots a_{s-1},b} \text{=0 → fix} \; F^{a_1 \cdots a_{s-1},b_1 b_2} \text{=0 → ... → fix} \; F^{a_1 \cdots a_{s-1},b_1 \cdots b_{s-1}}$

▪ This step also gives EoM → difficult to disentangle **EoM** and **Constraints**

▪ Fradkin Vasiliev construction $\qquad S = \int \sum_{r=0}^{s-1} \frac{a_r}{\Lambda^r} \left(F^{(s-1,r)}\right)^2$

✓ Inconsistent for **Quartic Order**

Unfolding

❖ Universal treatment of **EoM** & **Constraints**

❖ No privilege to metric

❖ Gravity ex.

Fields: e^a | ω^{ab} | $C^{ab,cd}$ | $\cdots$

Eqns: $De^a = 0$ | $D\omega^{ab} = e_c\, e_d\, C^{ac,bd}$ | $D\, C^{ab,cd} = e_e\, C^{abe,cd} + \cdots$ |
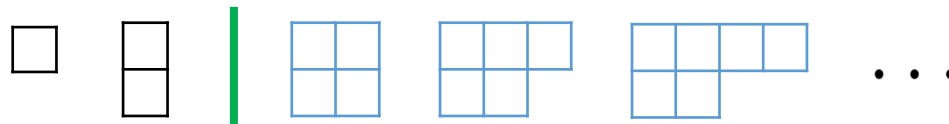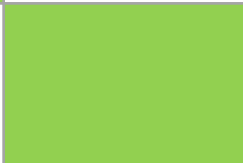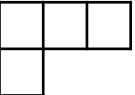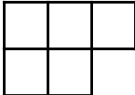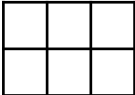
1-form **0-form**

❖ Spin s
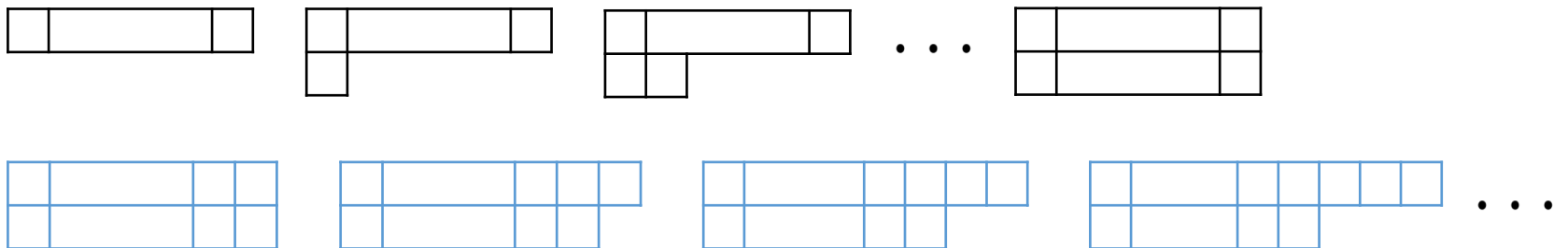
0-form field C_I **Twisted Adj** rep generated by K^I

1-form field A^I **Adjoint** rep of HS algebra generated by K_I

❖ Unfolded equations for HS gravity

$$d\,A^I + f^I_{JK}(C)\,A^J\,A^K = 0$$

$$d\,C_I + g^J_{IK}(C)\,C_J\,A^K = 0$$

■ Integrability condition → impose conditions on $f^I_{JK}(C)$ and $g^J_{IK}(C)$

➤ $f^I_{JK}(0)$: HS algebra structure constant

➤ $g^J_{IK}(0)$: Twisted Adjoint Representation

➤ Infinite dimensional Lie Algebroid

■ Vasiliev identified on $f^I_{JK}(C)$ and $g^J_{IK}(C)$ up to $O(C^3)$ ['88, '89]

Vasiliev's Equation in 4d

❖ HS algebra realized by oscillators $Y_A Y_B$, $Y_A Y_B Y_C Y_D$, …

$$A^I \quad \rightarrow \quad A(Y) \qquad\qquad C_I \quad \rightarrow \quad C(Y)$$

❖ Doubling of oscillator space: $A(Y,Z)$, $C(Y,Z)$, $S_A(Y,Z)$

new fields

❖ The Equations

$$dA + A \star A = 0 \qquad dC + [A \overset{\star}{,} C] = 0 \qquad dS_\alpha + [A \overset{\star}{,} S_\alpha] = 0$$

Algebraic constraints
$$[C \overset{\star}{,} S_\alpha] = 0 \qquad [S_\alpha \overset{\star}{,} S_\beta] = \epsilon_{\alpha\beta}(1 + e_\star^{i\,\Theta_\star(C)} \star C)$$

❖ Z dependent part is yet another set of auxiliary fields

- Solve the equation along Z direction → Recover the unfolded eq.

- Homotopy integral

Issues of Non-Locality

❖ String Theory

- HS gravity as tensionless string → highly non-local

❖ Bottom-Up Approach

- Cubic interaction: Local

- Quartic interaction (holographic reconstruction): Non-local

❖ Vasiliev equation

- Divergent 3pt amplitudes [Giombi-Yin 10, …]

- Divergent cubic interactions [Boulanger-Kessel-Skvortsov-Taronna 15]

Issues of Non-Locality

❖ Unfolded equation of HS

$$d\,A + A \star A + V_3(C, A, A) + V_4(C, C, A, A) + \cdots = 0$$

$$d\,C + A \star C - C \star A + W_3(C, C, A) + W_4(C, C, C, A) + \cdots = 0$$

❖ Perturbative expansion around AdS background

- Cubic: $\quad V_3(C, \eta, \Omega), V_4(C, C, \Omega, \Omega), W_3(C, C, \Omega),$

- Quartic: $\quad V_3(C, \eta, \eta), V_4(C, C, \eta, \Omega), V_5(C, C, \Omega, \Omega),$
  $$W_3(C, C, \eta), W_4(C, C, C, \Omega)$$

❖ Locality [Vasiliev et al 16~19]

- A restricted form of $\;V_n, W_n$

- Use of different homotopy → Local $\;V_3, V_4, W_3$

Interactions in Unfolded Equations

❖ Unfolded equation of HS

$$d\,A + A \star A + V_3(C,A,A) + V_4(C,C,A,A) + \cdots = 0$$

$$d\,C + A \star C - C \star A + W_3(C,C,A) + W_4(C,C,C,A) + \cdots = 0$$

❖ Deformation problem

- Deformation of ∞-dim Lie algebra to Lie algebroid

- Deformation of ∞-dim Poisson structure

- Deformation of L_∞-algebra and A_∞-algebra

- A_∞-algebra deformation from a deformation of "extended" HS algebra [Sharapov-Skvortsov]

  → Generate all order interactions

Summary of my recent works

❖ HS Algebra

- Minimal Orbit, Howe Duality & Structure Constants
  [w/ Mkrtchyan 14, 16]

- Character Method [w/ Basile, Bekaert 18]

- Curious Observation:

  Conformal HS Gravity = HS Algebra [w/ Basile, Bekaert 18]

❖ On-going works

- Unfolded Equation & Action for 3d HS Gravity

- Unfolded Equation for Conformal HS Gravity
  - Issues of (non-)locality

- Non-minimal Coadjoint Orbits & Its Quantization
  (and its relation to deformation of HS algebra)