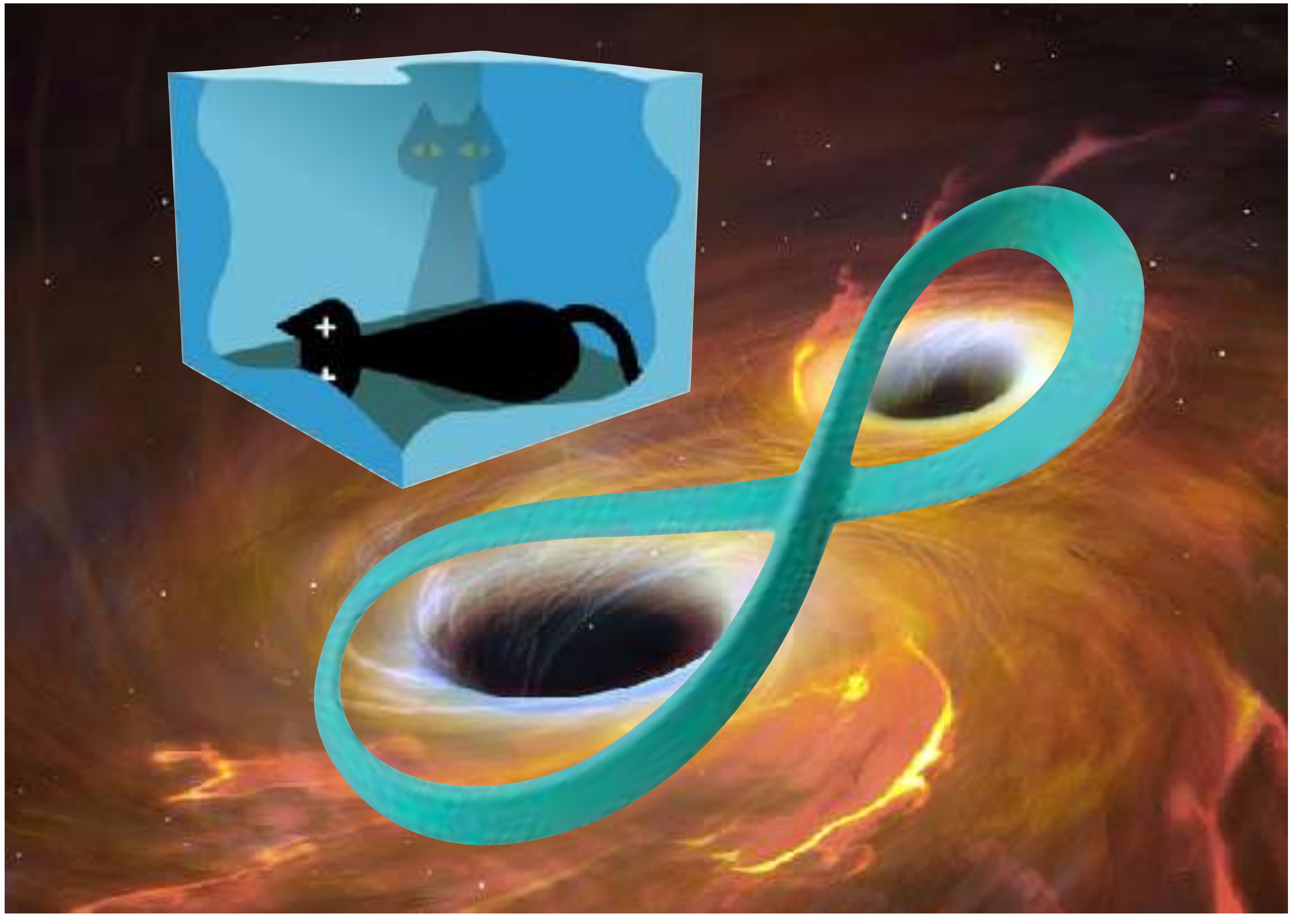
양자, 중력 그리고 무한대

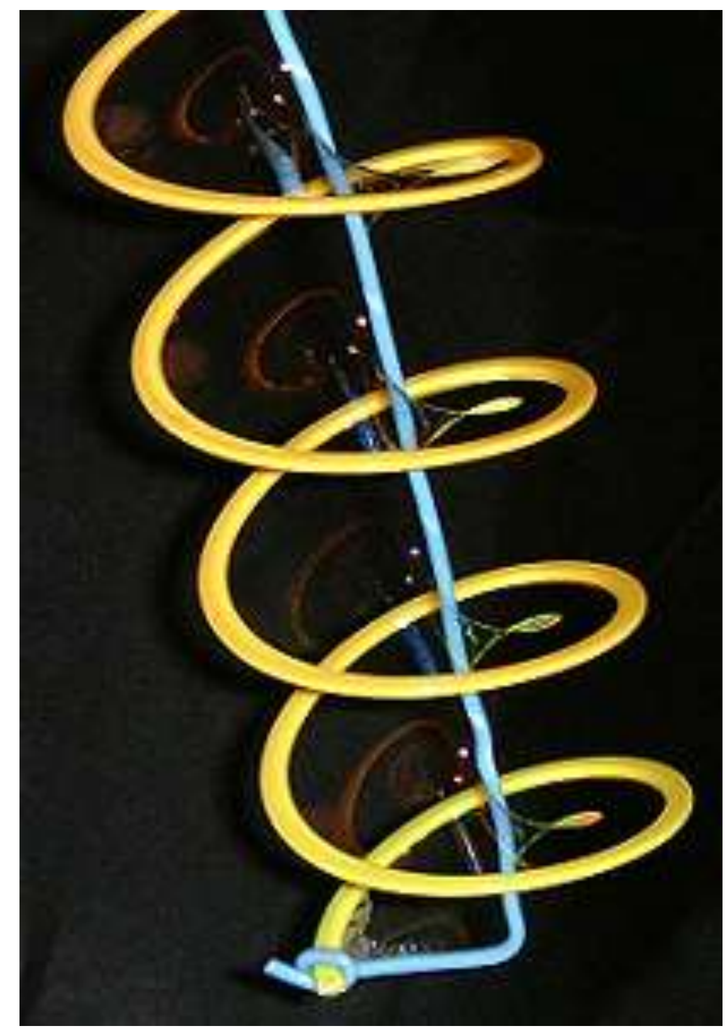


정의헌 (물리학과)

Quantum Mechanics

Variational Problem

Lagrange

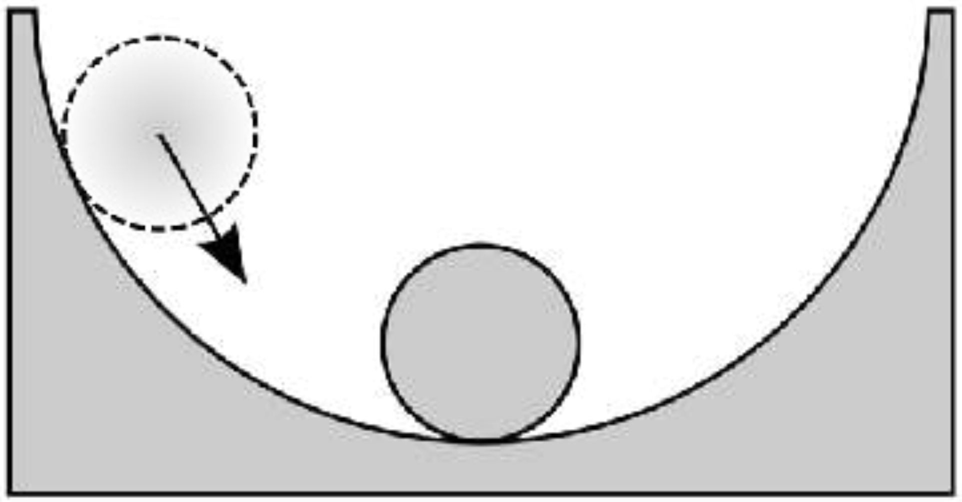


Minimal Surface

Variational Problem

- Newton's equation from variational problem

➡ Euler-Lagrange equation

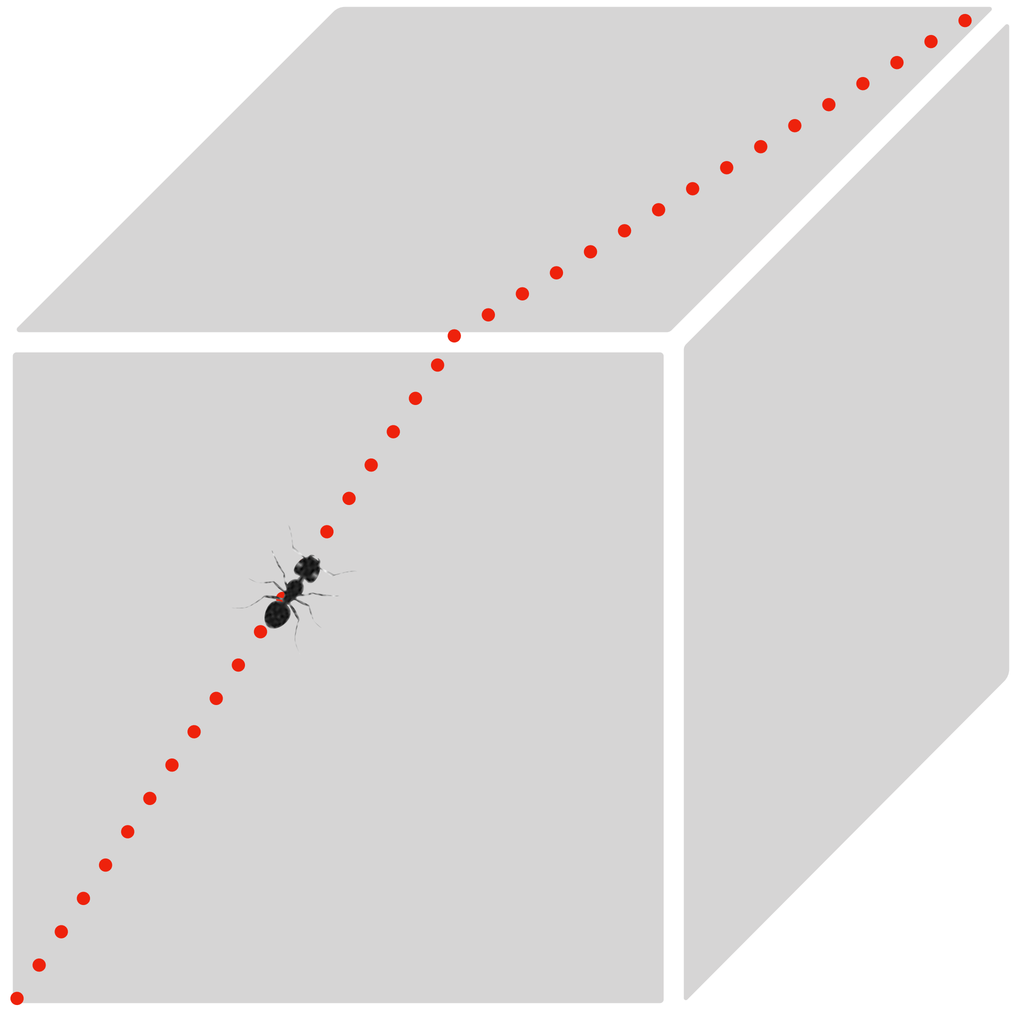


Minimize (or maximize)

[(Potential Energy) - (Kinetic Energy)] x (Time)

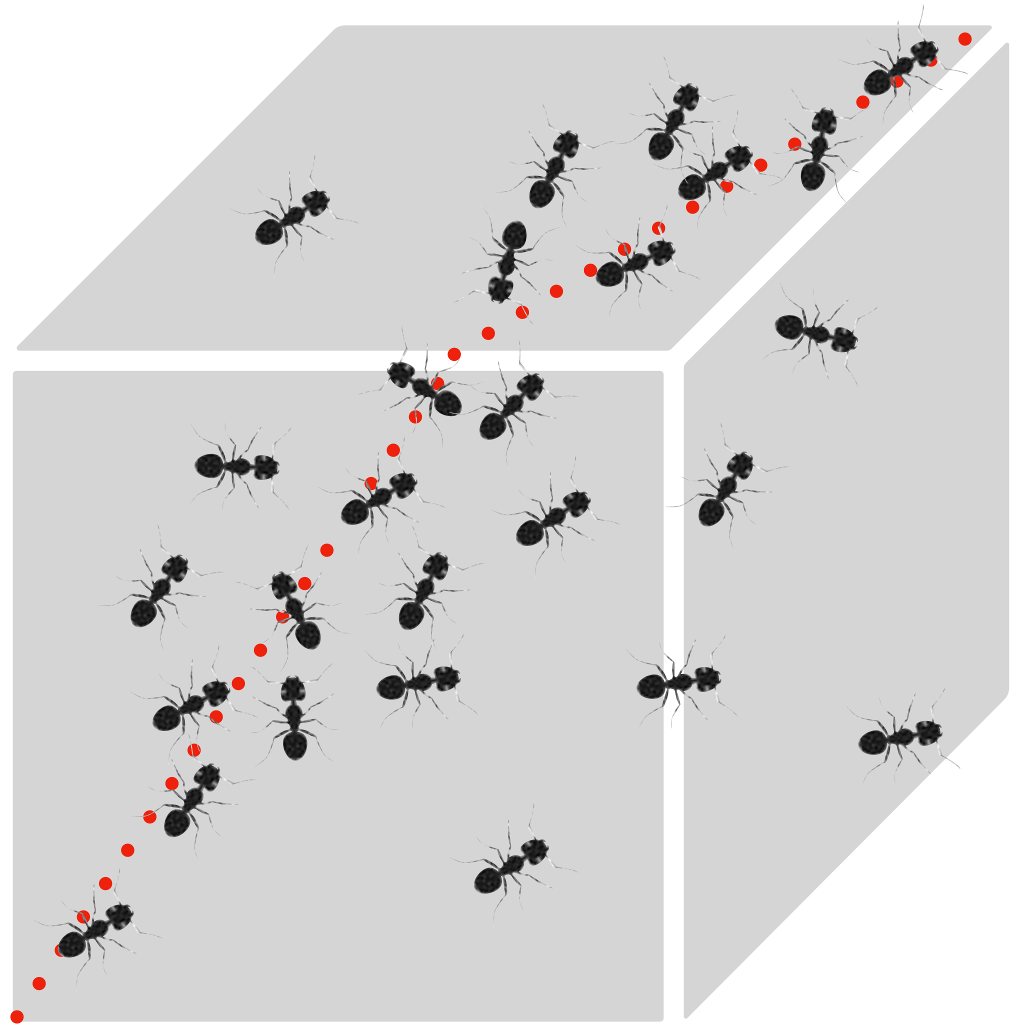
Action (Lagrangian) *S* = [Energy]x[Time]

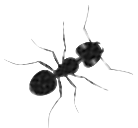
Shortest path



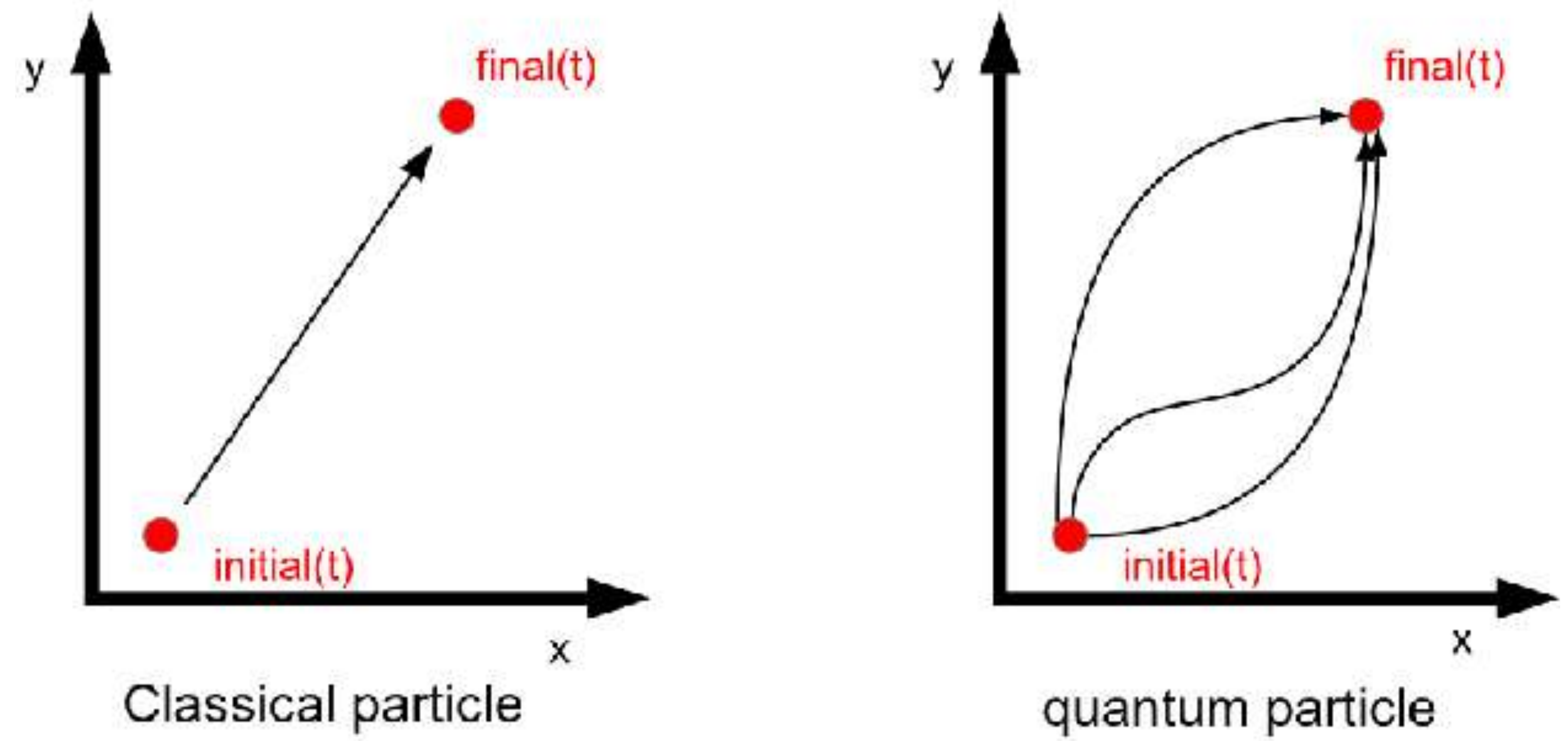
How an ant knows the shortest path?

Shortest path




Pheromone

Quantum Mechanics



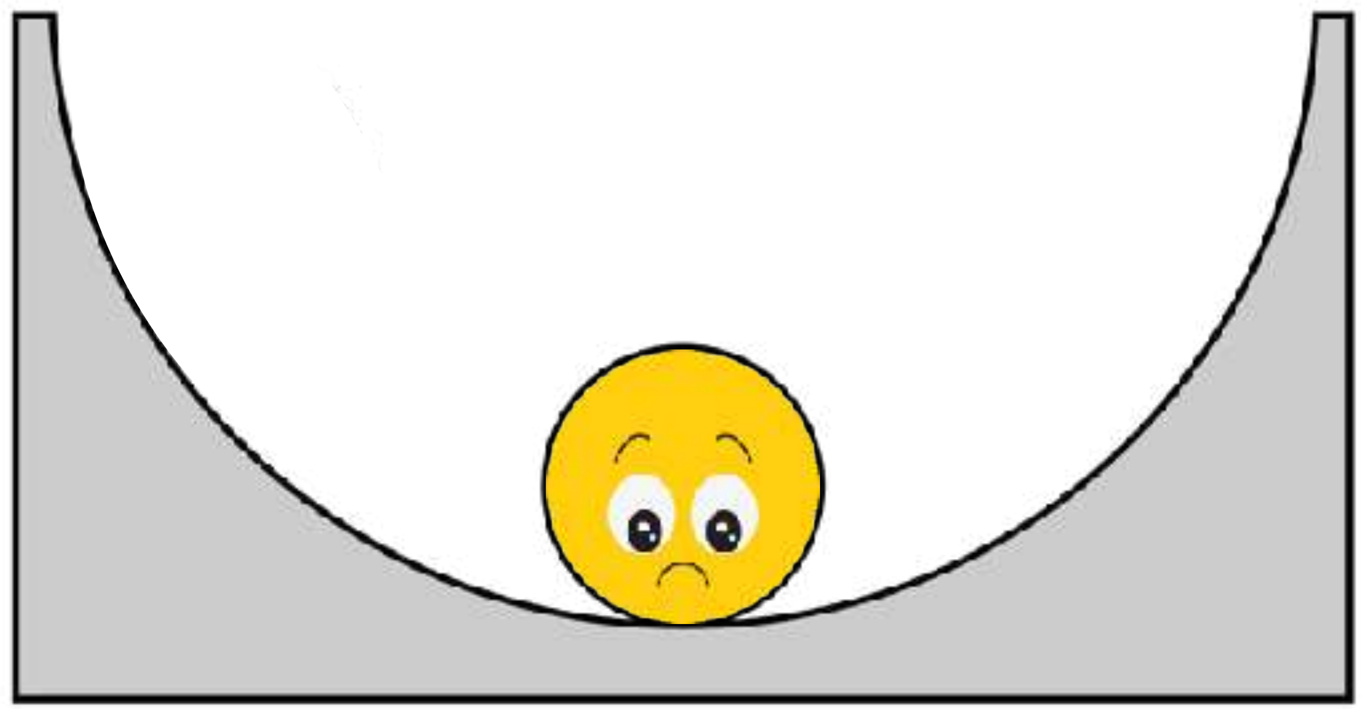
Sum over paths with weight $e^{\frac{i}{\hbar}S}$

(Feynman path integral)

Quantum Mechanics

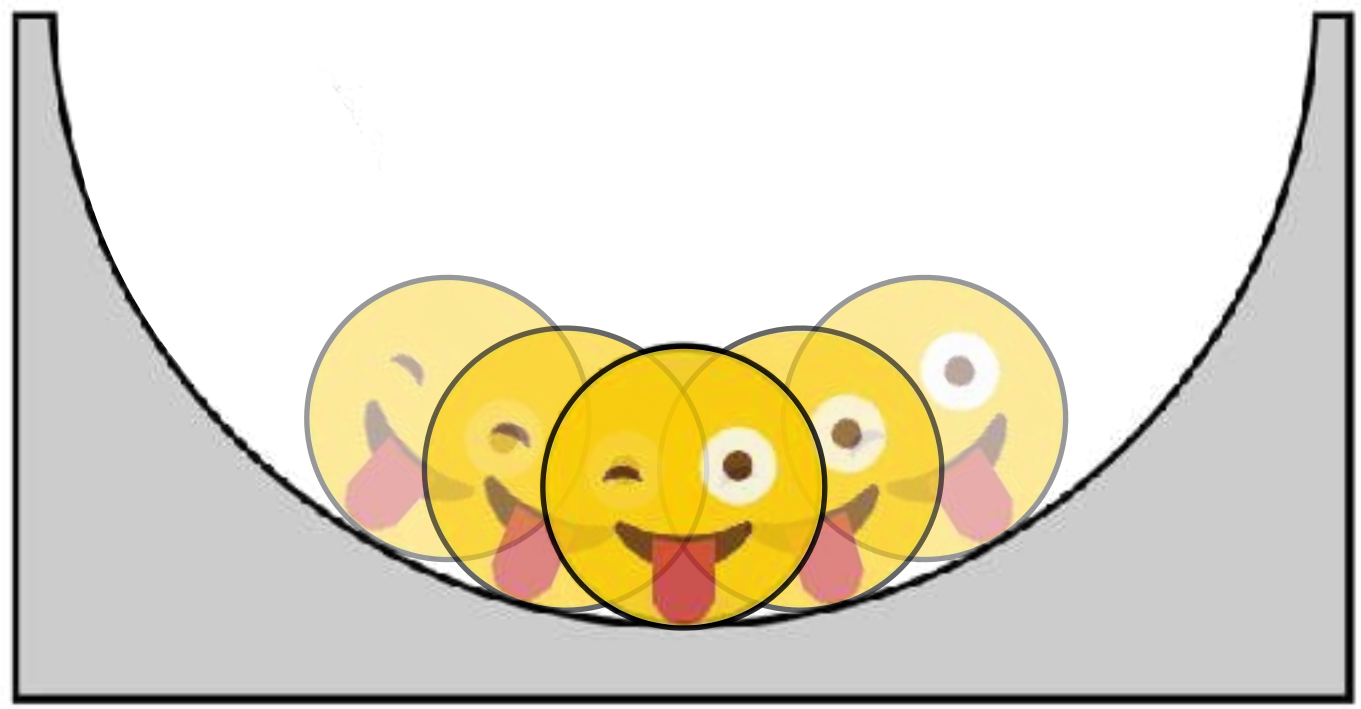
- Wave nature: $e^{\frac{i}{\hbar}S}$ → **Interference !**
- [Energy] $\Longleftrightarrow$ [Time] conversion ratio
    - [Energy]x[Time]=$\hbar$
- [Time] $\Longleftrightarrow$ [Length] conversion ratio
    - [Length]/[Time]=c
- [Mass] $\Longleftrightarrow$ [Length] conversion ratio
    - [Mass]x[Length]=$\hbar/c$

Classical Mechanics



Minimum energy is 0

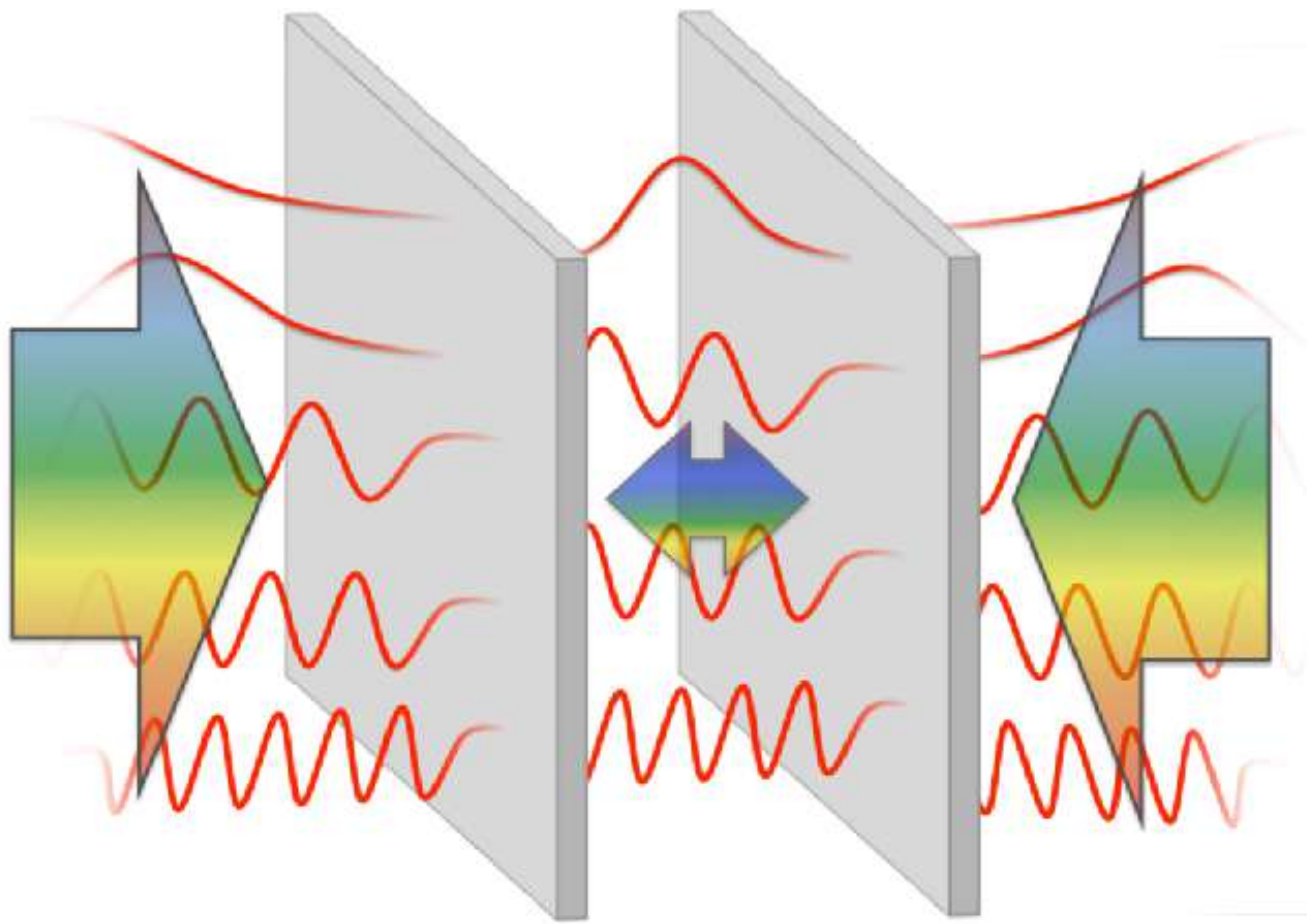
Quantum Mechanics



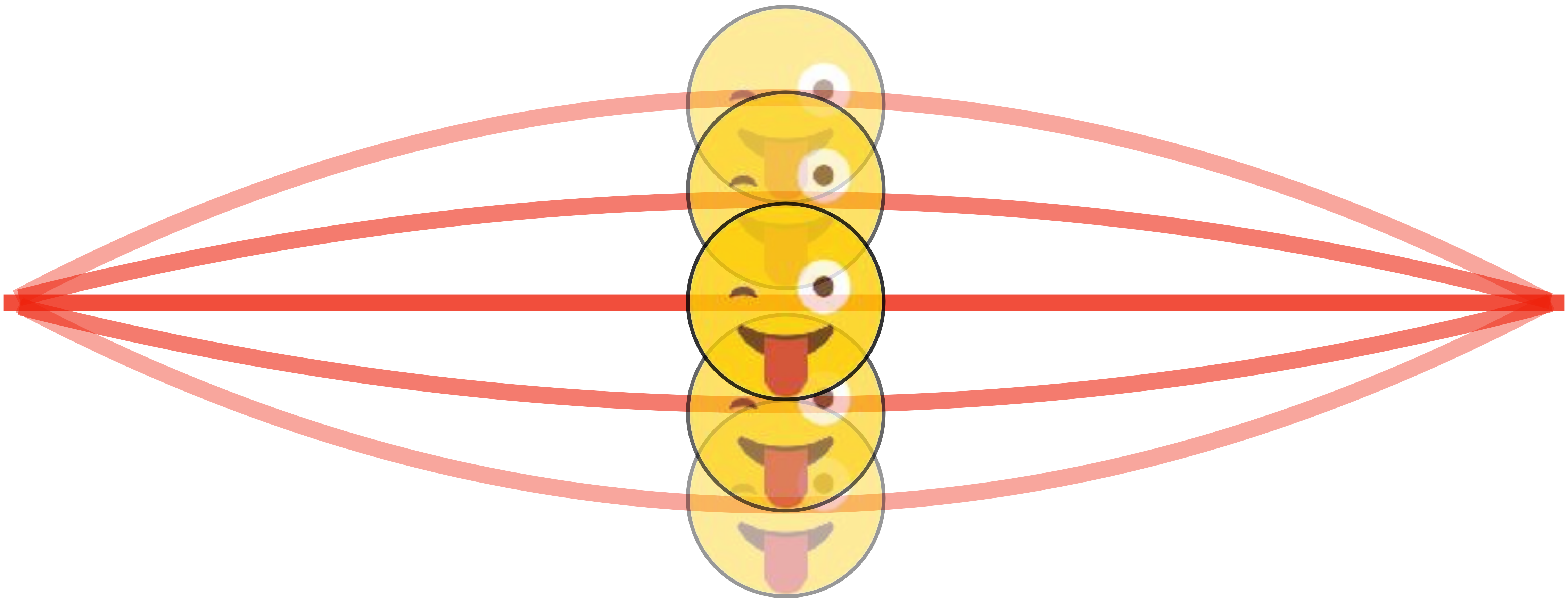
Minimum energy is not 0 !

Quantum EM field

Casimir Effect

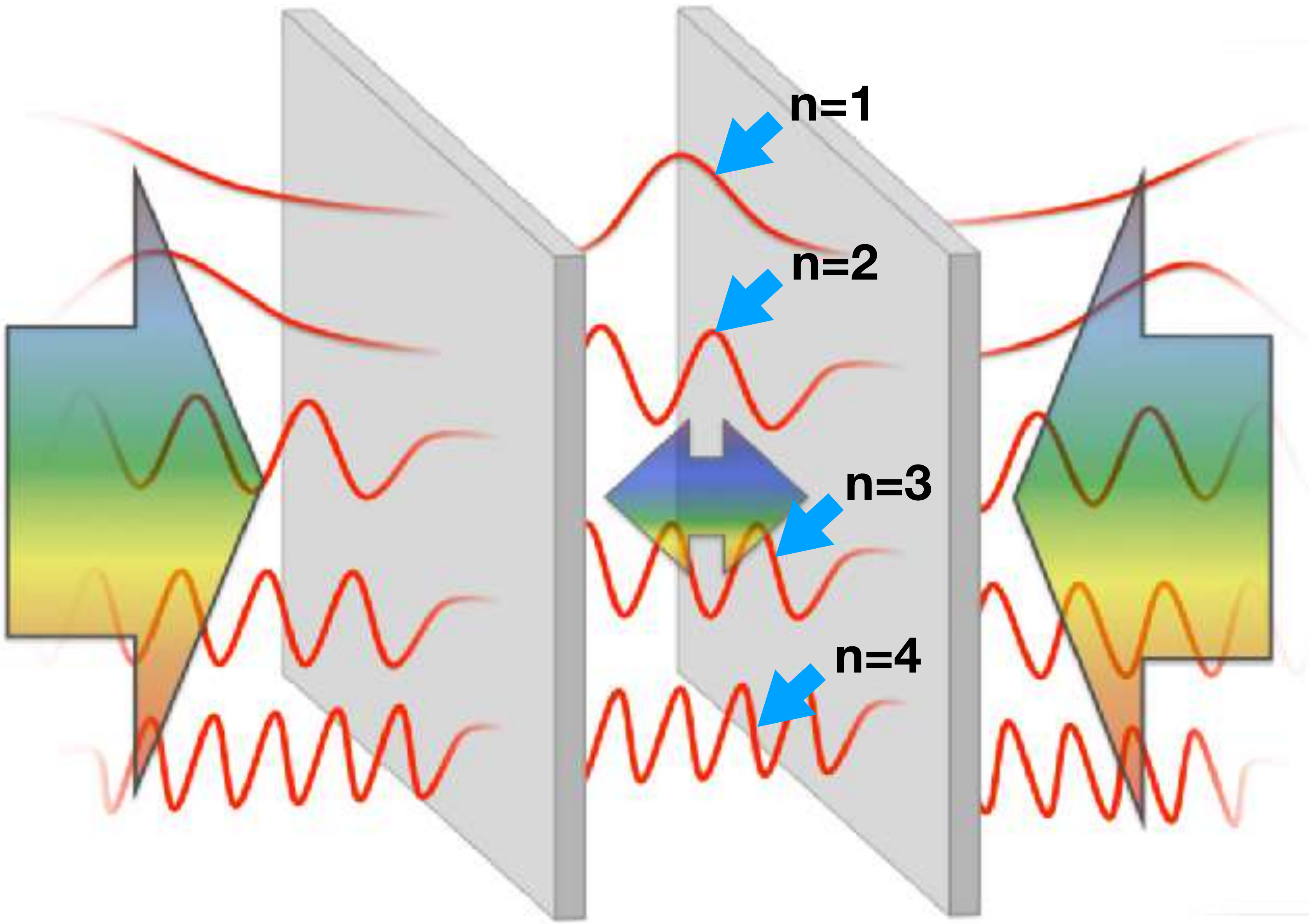


Casimir Effect



Minimum (vacuum) energy of each wave mode is not 0 !

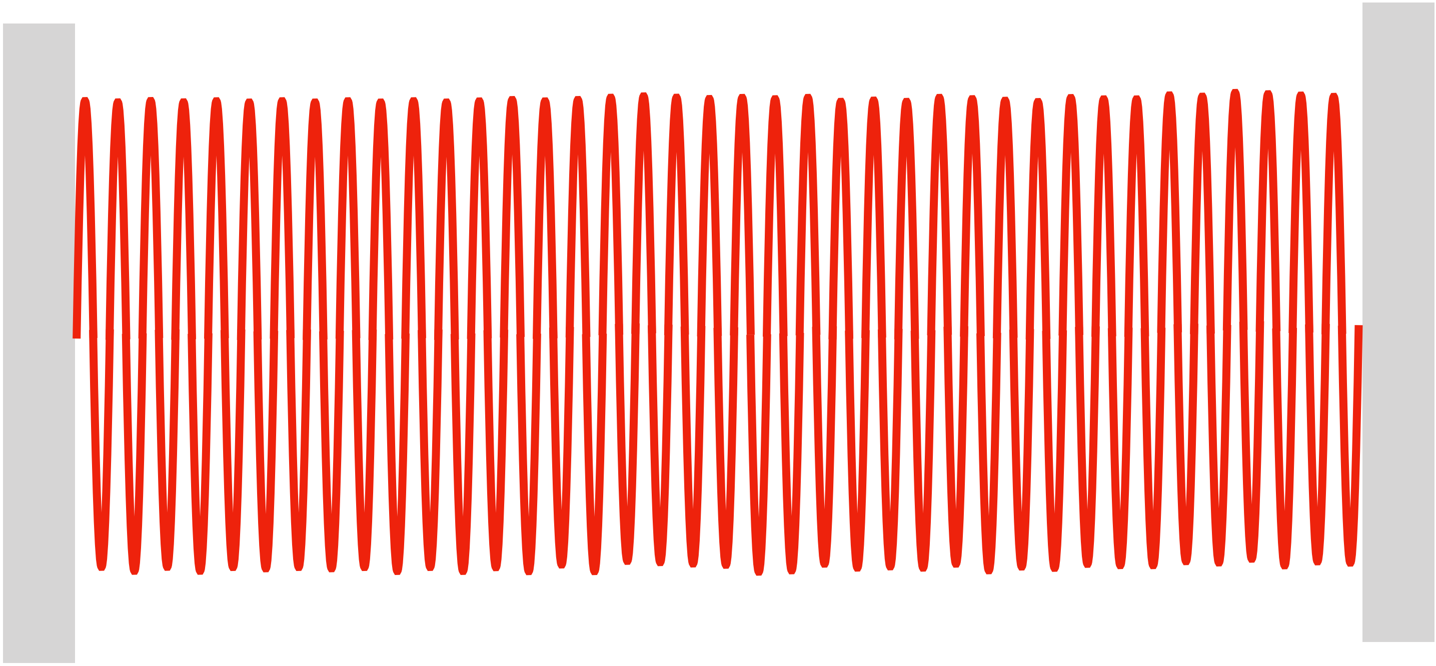
Casimir Effect



Infinitely many modes!

Casimir Energy

- Each mode has a finite vacuum energy $E_n^{\mathrm{vac}} = C\,\hbar\,n$
- Total vacuum energy: $E_{\mathrm{tot}}^{\mathrm{vac}} = \sum_{n=1}^{\infty} E_n^{\mathrm{vac}} = C\,\hbar \sum_{n=1}^{\infty} n$
- Infinity occurs at 0 distance (or ∞ energy)



Ramanujan summation

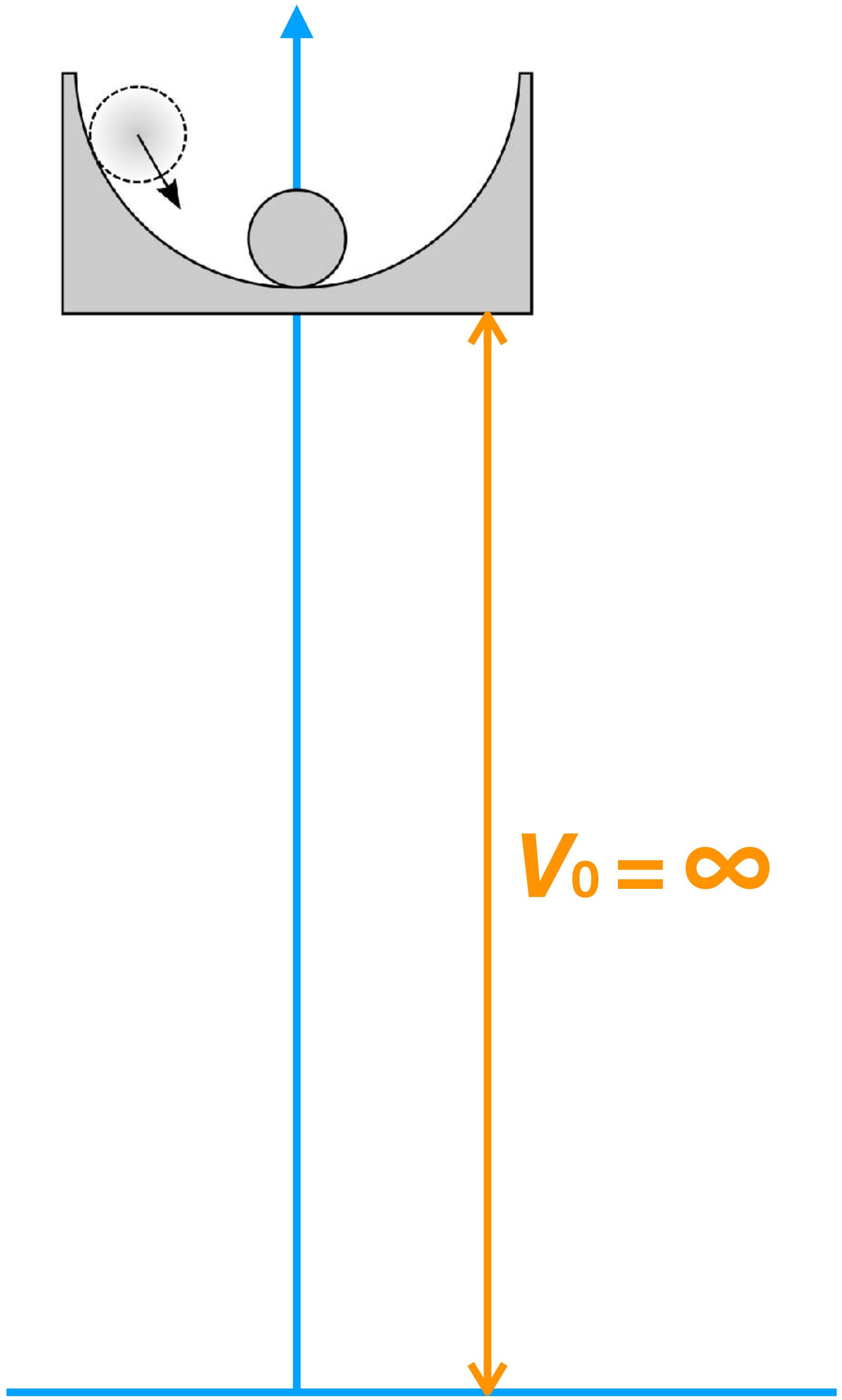
$$\begin{aligned} c &= 1 + 2 + 3 + 4 + 5 + 6 + \cdots \\ 4c &= \quad\;\; 4 \quad\;\; + 8 \quad\;\; + 12 + \cdots \\ c - 4c &= 1 - 2 + 3 - 4 + 5 - 6 + \cdots \end{aligned}$$

$$\frac{1}{(1-x)^2} = 1 + 2\,x + 3\,x^2 + 4\,x^3 + \cdots$$

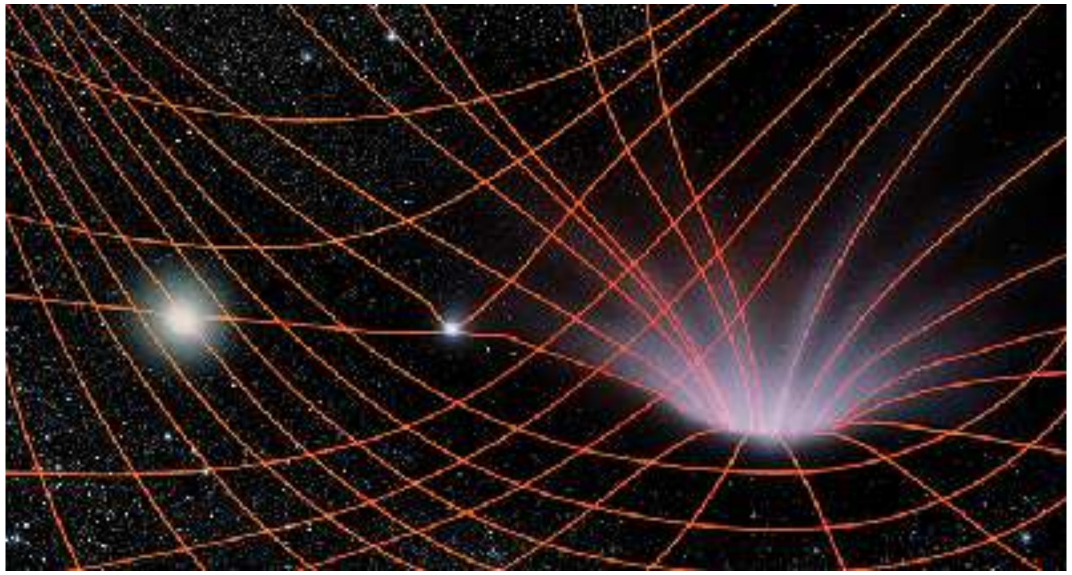
$$\boxed{1 + 2 + 3 + 4 + \cdots = -\frac{1}{12}}$$

Consistent with observation!

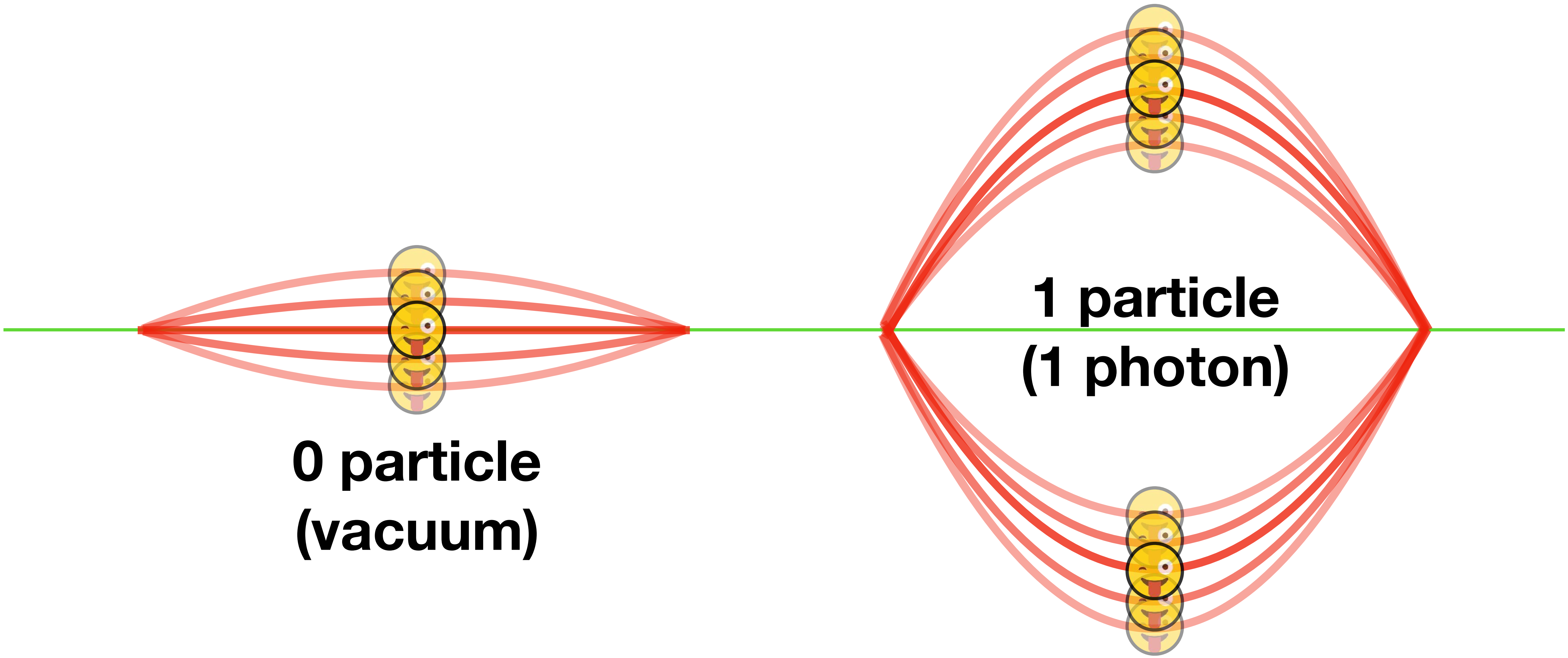
Where has ∞ gone?



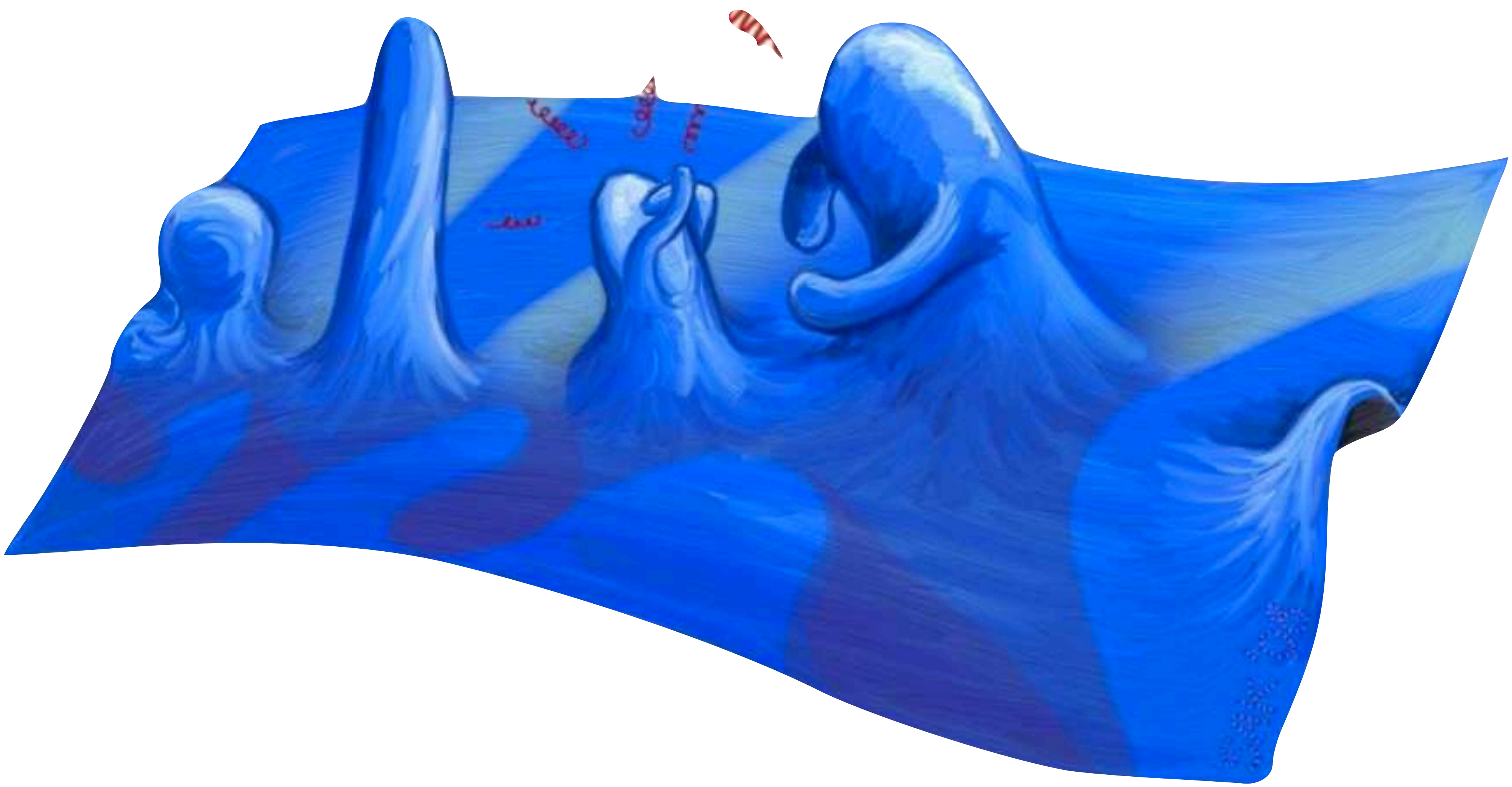
- ∞ shift of zero-point energy
- zero-point energy: mere constant?
- It affects gravity!



Stronger EM field



Quantized Field → Particle



Interacting fields

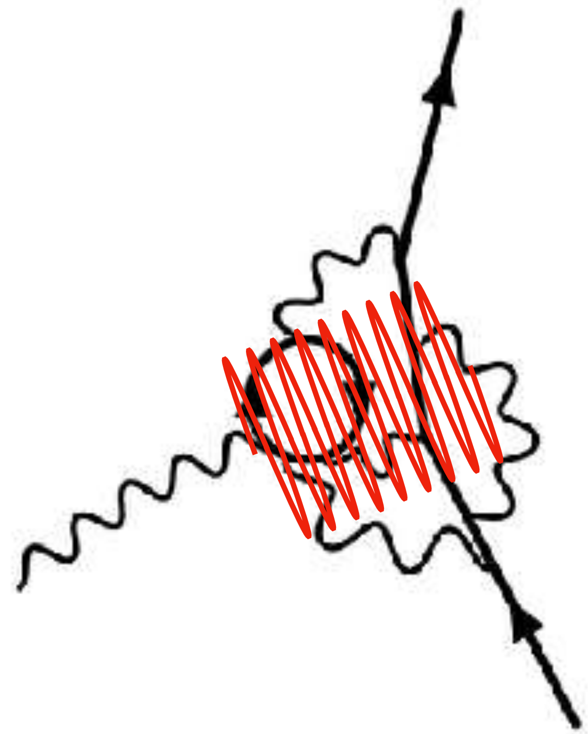
Photon

W boson

Electron

∞ from interaction

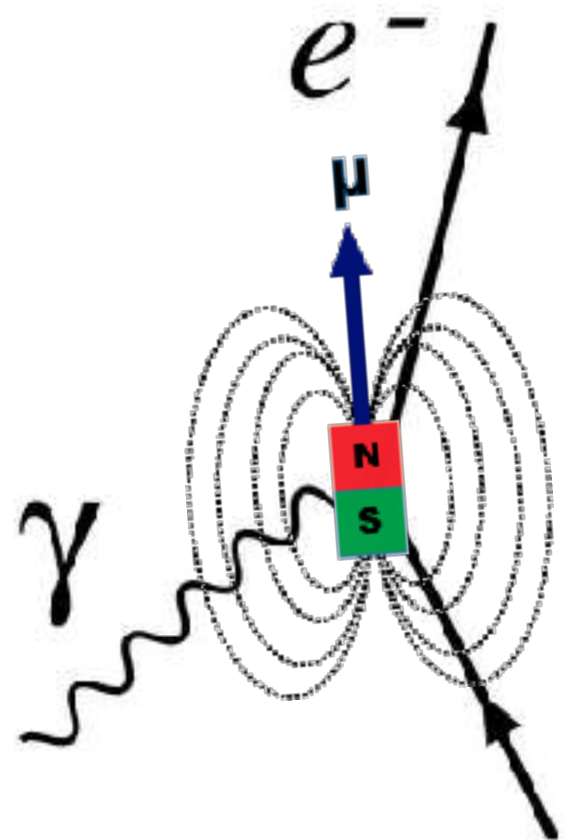
Classical Effect



Quantum Effect

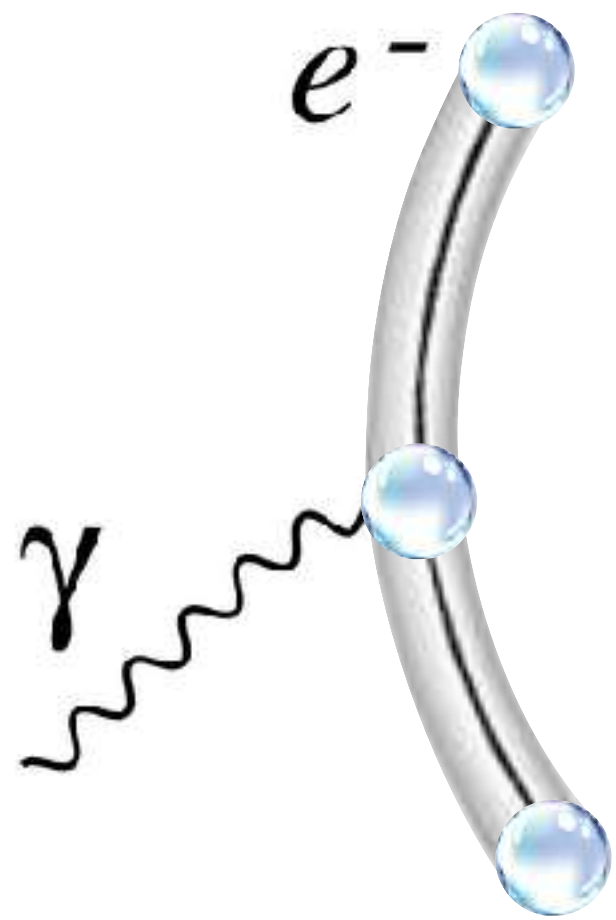
- ∞ occurs because particles have zero size
- Remove ∞ by ∞ shift of charge and mass (Renormalization)

∞ from interaction



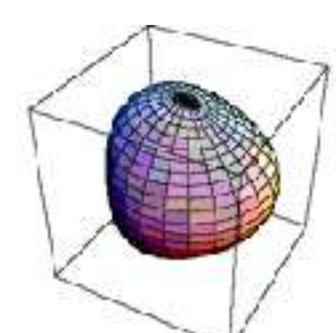
- g-factor (magnetic dipole moment)
  - $g_{experiment}$ = 2.002 331 841 16(13)
  - g_{theory} = 2.002 331 836 20(86)

Dirac’s bubble electron



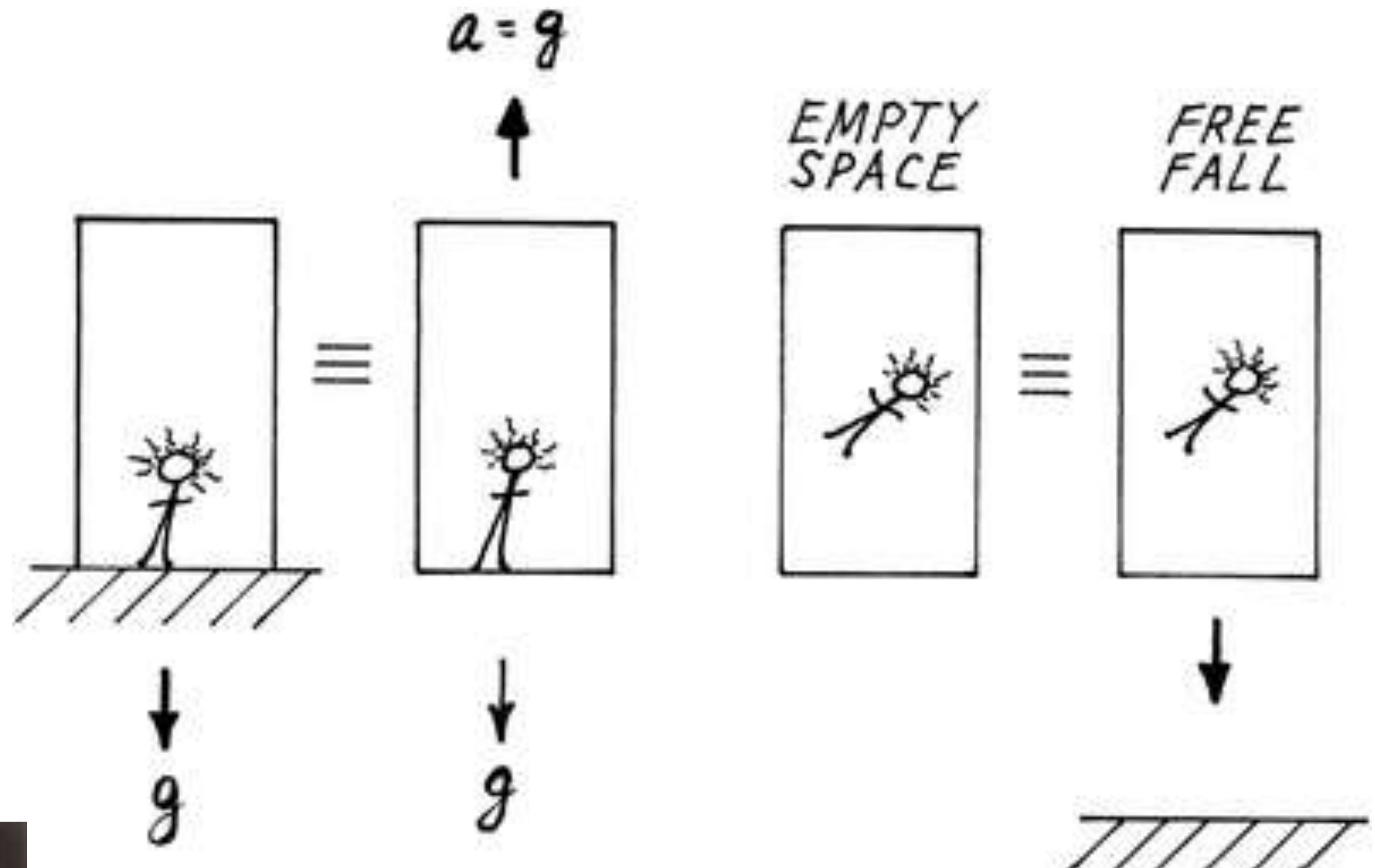
- Bubble has a finite size (a possible remedy of ∞)

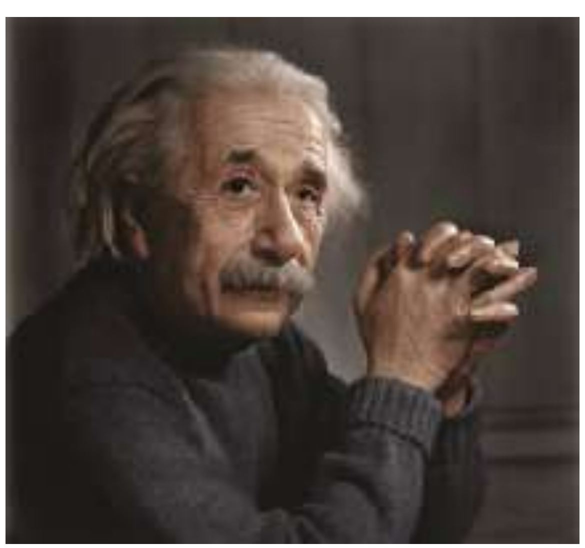
- Excitations as different particles



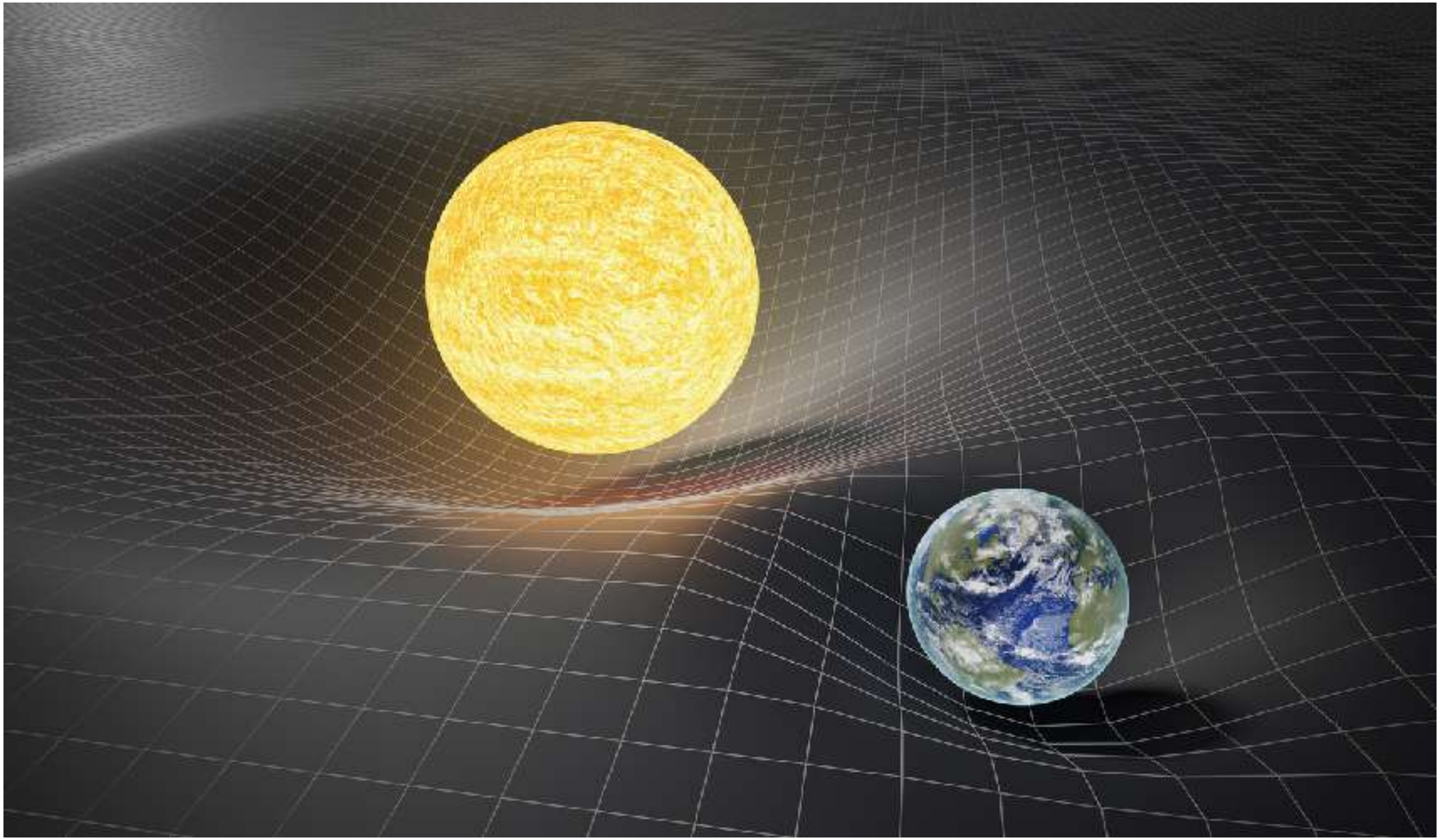
Gravity

General Relativity



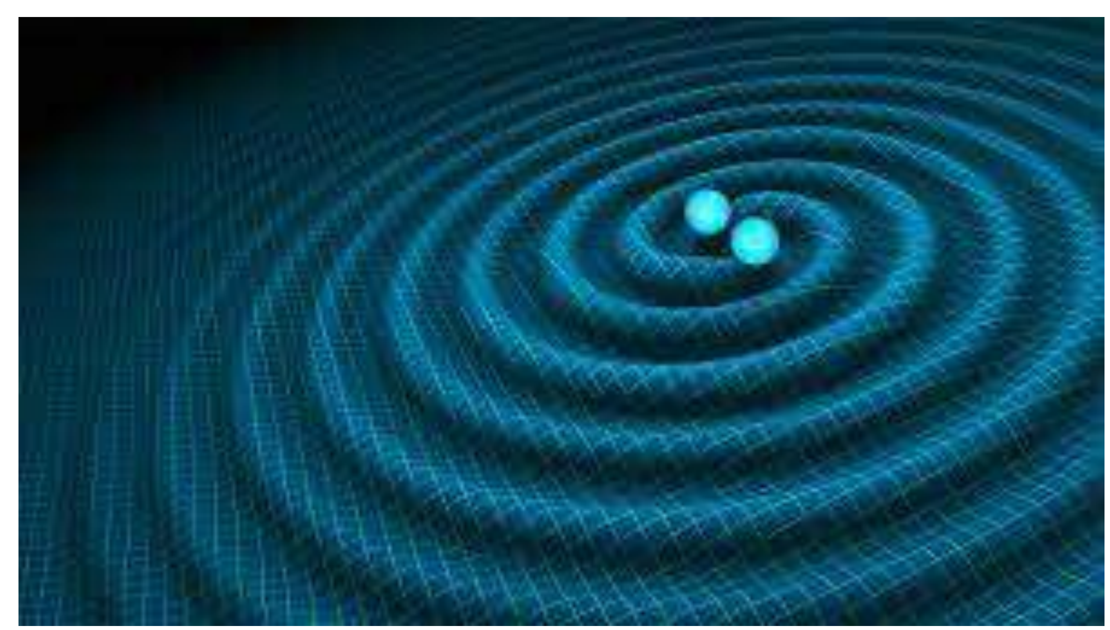


General Relativity



General Relativity

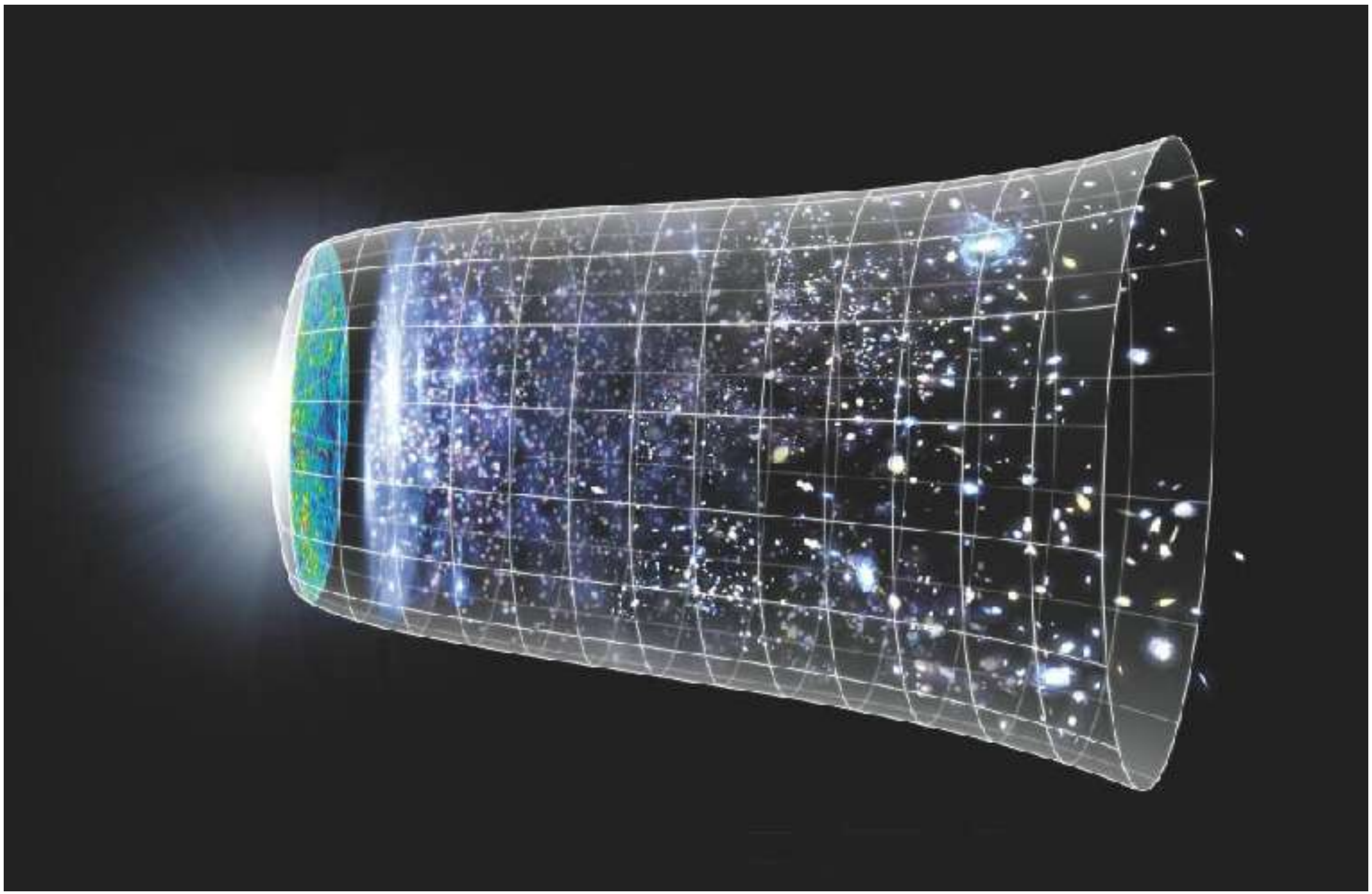
- Field: metric of spacetime
  - Tells how much spacetime is curved
  - Non-trivial dynamics
  - Quantization ➔ Graviton



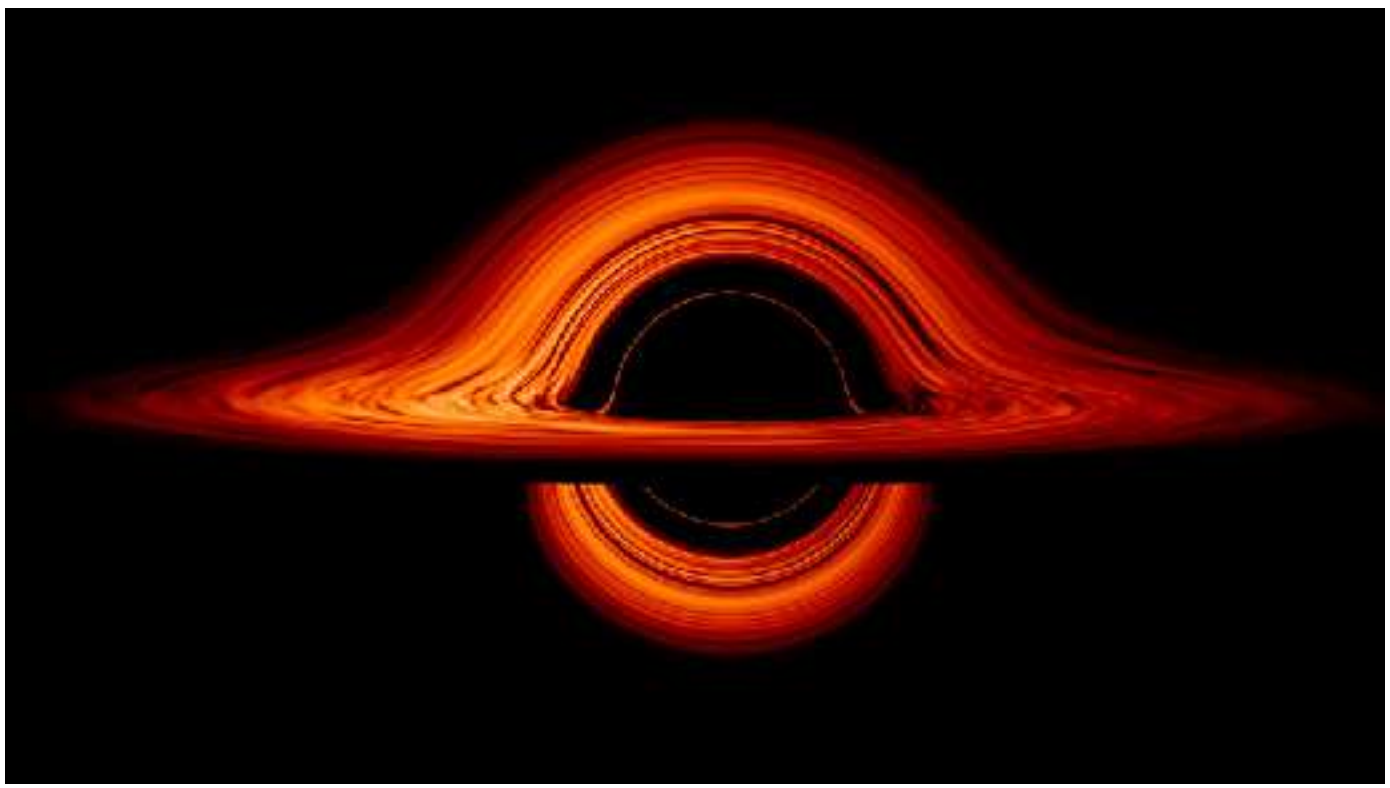
Singularities

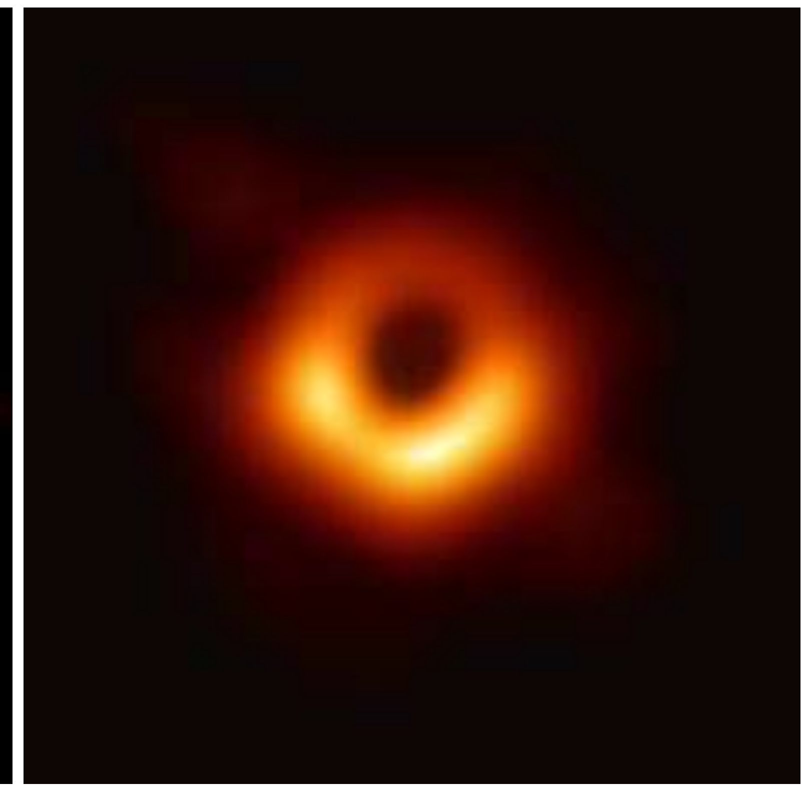
- Singularities(∞) are inevitable in General Relativity
  - Black hole, Big Bang
  - Suggest Incompleteness of GR

Big Bang



Black Hole





$$\frac{1}{2}\, m\, c^2 = G_N\, \frac{M\, m}{R_H} \quad \Rightarrow \quad R_H = \frac{2\, G_N\, M}{c^2}$$

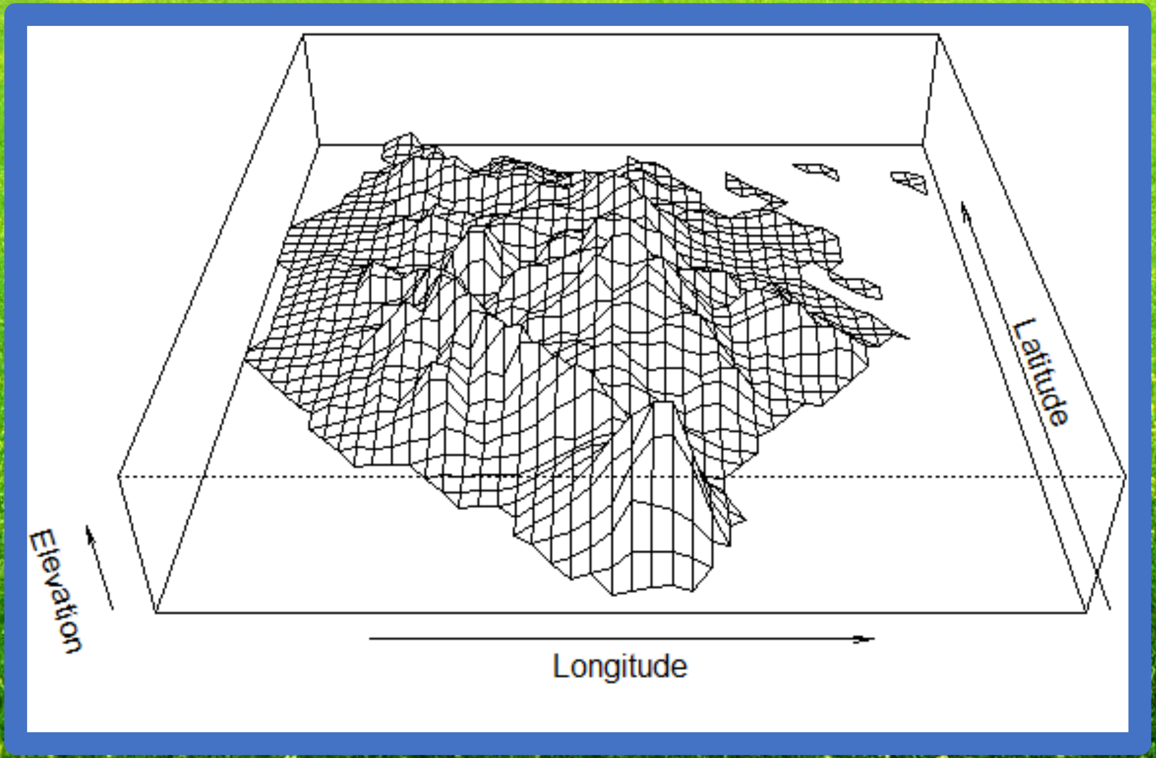
Quantum Gravity

Gravity as a field

Photon

W boson

Electron

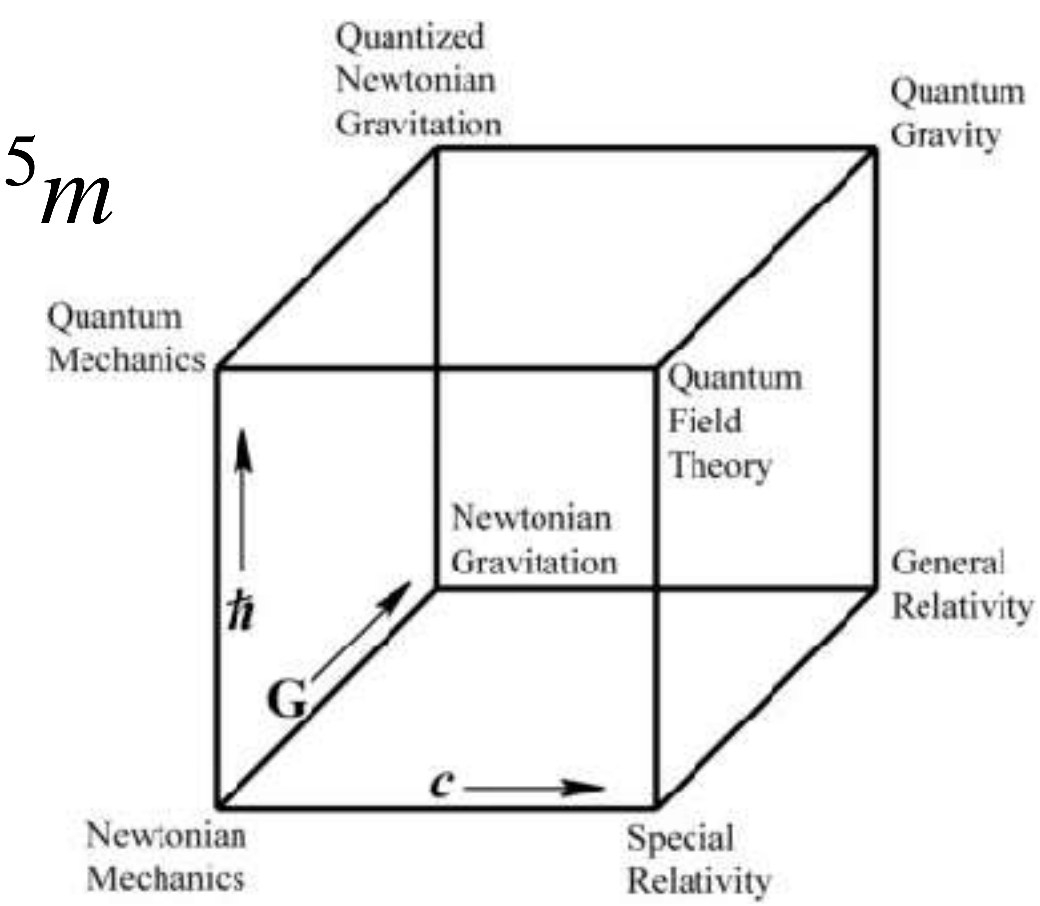


Graviton

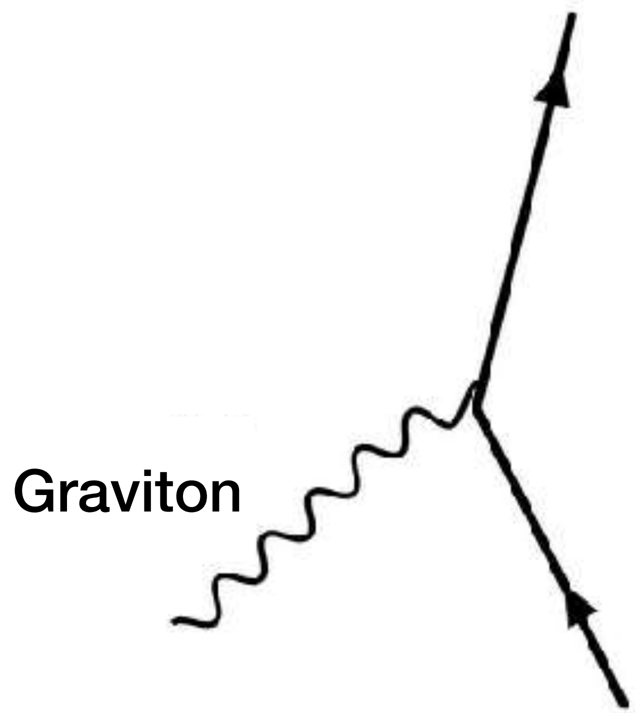
Quantizing Gravity

- QM ➔ [Length]x[Mass]=$\hbar/c$
- Gravity ➔ [Length]/[Mass]=G_N/c^2
- Quantum Gravity has a length scale (Planck length)

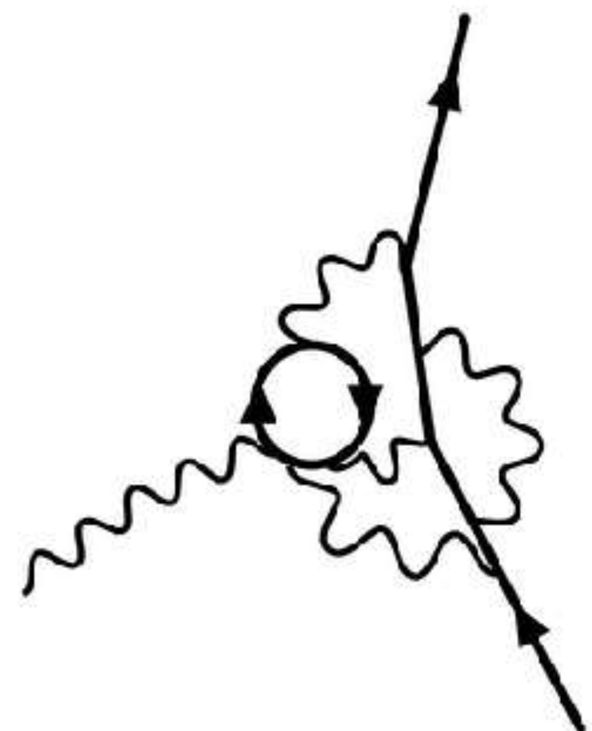
$$\ell_P = \sqrt{\hbar\, G_N/c^3} \simeq 1.6 \times 10^{-35} m$$



Quantizing Gravity



Classical Effect

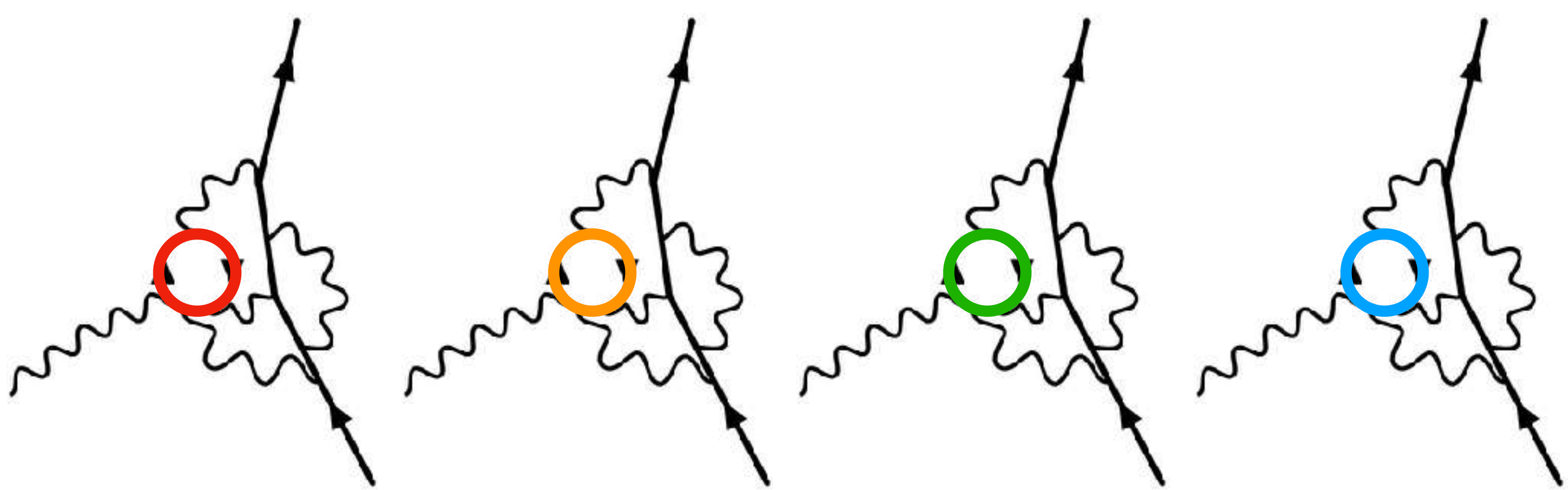


Quantum Effect

- Quantum effects: $(\ell_P E)^2, (\ell_P E)^4, \cdots$
- ∞ numbers of ∞ ➔ ∞ number of shifts (∞ new parameters)
- GR should be modified at high energy

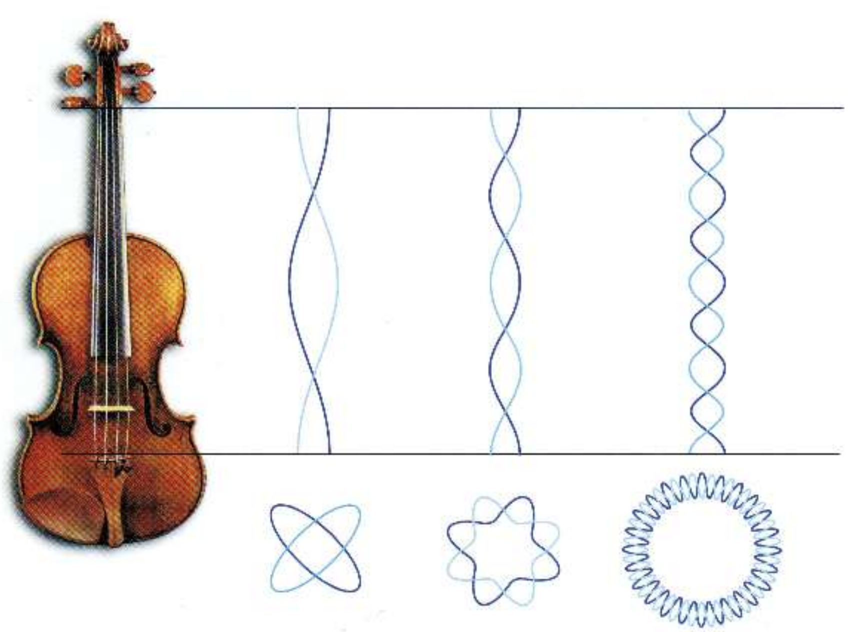
Quantizing Gravity

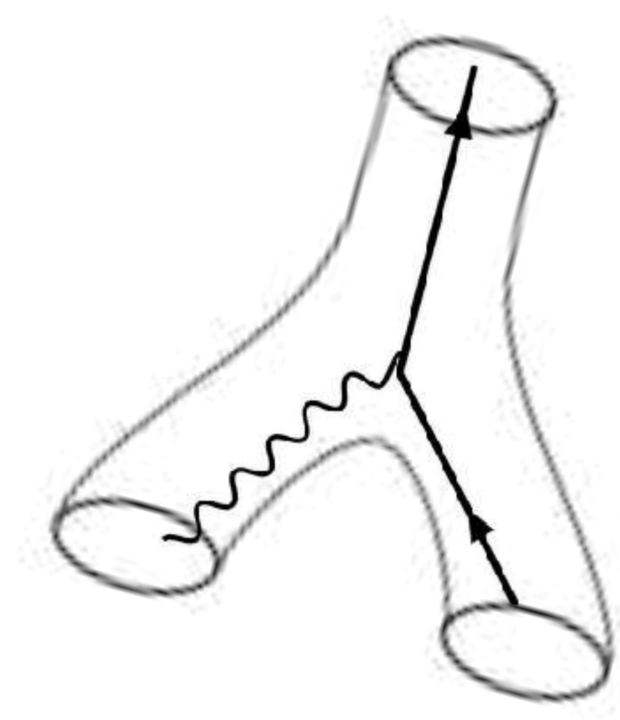
- Quantum effects arise from virtual particles
- Summation over virtual particles
  - Typically, ∞ add up
  - Sometimes, ∞ partially cancels (Super Gravity)



String Theory

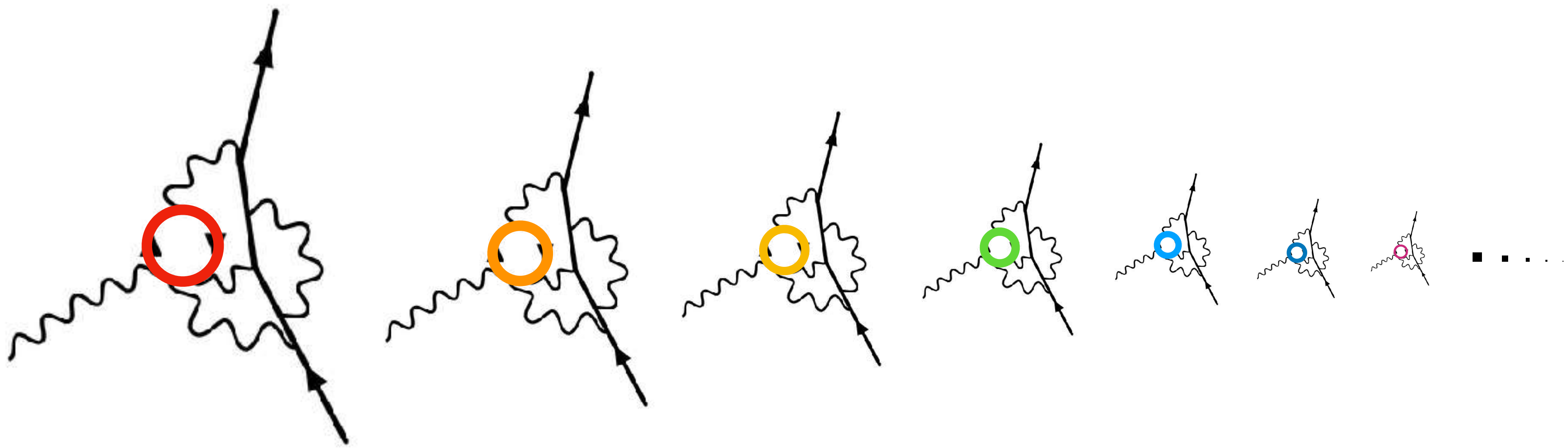
- String has a finite size ➔ No ∞ arises
- Excitations of string as different particles
  - Include massless particles like photon, **graviton** etc
  - ∞ amount of massive particles





String Theory

- ∞ from ∞ amount of virtual particles completely cancel out
- ∞ + 2∞ + 3∞ + … = 0



Massless Particles

- Massless particles are special
  - Spin 1 massless particles ➔ Gauge theory
  - Spin 2 massless particles ➔ Gravity
- What about higher spin massless particles?
  - a single spin>2 massless particle is not consistent
  - In fact, we need ∞ many higher spin massless particles

Higher Spin Gravity

- An extension of GR with ∞ higher spin particles
- Non-locality (finite size) like in String Theory
- Complete cancelation of ∞ in its vacuum energy

  ➡ A candidate for Quantum Gravity
- Interesting mathematical structure